Physics I

Table of Contents

Equations of Motion

$$a = constant$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u+v)t \Rightarrow \bar{v} = \frac{u+v}{2}$$

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v(t) = v_0 + at$$

$$v^2(x) = v_0 + 2a\Delta x$$

Tsiolkovsky Rocket Equation

$$v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

v_e is the exhaust speed
m_0 is the initial mass
$m(t)$ is the function of mass over time
g is gravitational acceleration, if present

Newton's Laws of Motion

First Law (Law of Inertia)

Every object continues in its state of rest, or of uniform velocity, if no net force acts on it.

Second Law ($\vec{\boldsymbol{F}} = \boldsymbol{m}\vec{\boldsymbol{a}}$)

The acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to the object's mass. The direction of acceleration is in the direction of the net force acting on the body.

Third Law (Principle of Action and Reaction)

Whenever an object exerts a force on a second object, the second object exerts on the first object an equal force in the opposite direction to the first.

Friction

$$F_{kf} = \mu_k F_N$$

$$F_{sf} \leq \mu_s F_N$$

Gravity

$$F = G\frac{m_1 m_2}{r^2} \Rightarrow g = G\frac{m}{r^2}$$

$$U(\infty) = 0$$

$$U(r) = -\frac{GMm}{r}$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

When close to the surface of the planet:

$$E = K + U = \frac{1}{2}mv^2 - \frac{1}{2}mv_{esc}^2$$

Lagrange Point

A point in the vicinity of the Earth orbits the Sun where a small satellite can orbit the Sun with the same period T as Earth's. This is possible because the Earth's gravitational pull allows for a slower orbit time to counteract the gravity from the Sun.

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

Above, M is the mass of the planet, r is the initial distance from the center of the planet. For Earth,

$$v_{Eesc} = \sqrt{\frac{2GM_E}{r_E}} \approx 1.12 \times 10^4$$

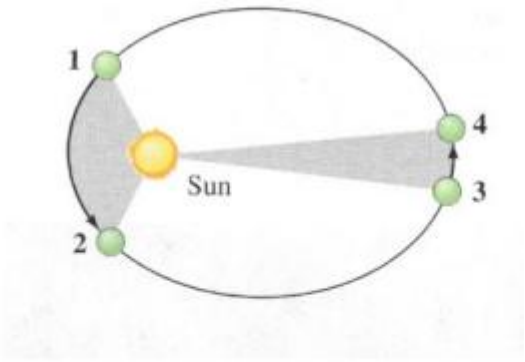
Kepler's Laws

First Law

The path of each planet around the Sun is an ellipse with the Sun at one focus.

Second Law

Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time.



Third Law

The ratio of the squares of the periods of any two planets revolving around the sun is equal to the ratios of the cubes of their semi-major axes. (Semi major axis is the greatest distance from the center of the ellipse)

$$\frac{s_1^3}{T_1^2} = \frac{s_2^3}{T_2^2}$$

Work & Energy

$$W = \Delta E = Fd\cos\theta = \vec{F} \cdot \vec{d}$$

$$W = \int_a^b \vec{F} \cdot \overrightarrow{dl} = \int_a^b F\cos\theta \ dl$$

Conservative forces are reversible, non-recoverable forces are not. Potential energy (U) is conservative.

$$U_g = mgh$$

$$U_{el} = \frac{1}{2}kx^2$$

$$F = -\frac{dU}{dx}$$

$$E = K + U \Rightarrow \text{Mechanical Energy} = \text{Kinetic E.} + \text{Potential E.}$$

$$P = \frac{dW}{dt} = \frac{dE}{dt} = \vec{F} \cdot \vec{v}$$

Hooke’s Law

$$F_s = -kx$$

$$W = \frac{1}{2}kx^2$$

Kinetic Energy

$$E_k = \frac{1}{2}mv^2$$

Work-Energy Principle: the net work done on an object by the net resultant force is equal to the change in kinetic energy of the object:

$$W = \Delta \mathrm{E_k} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}m(v_2^2 - v_1^2)$$

Circular Motion

Centripetal Acceleration

$$a_R = \frac{v^2}{r} = \omega^2 r$$

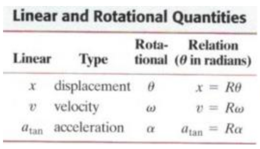
$$\omega = 2\pi f = \frac{d\theta}{\mathrm{dt}} = \frac{v}{r}, \qquad f = \frac{1}{T}$$

$$\alpha = \frac{d\omega}{dt}$$

$$1rad \approx 57.3°$$

$$v = R\omega, \qquad \mathrm{a_{tan}} = R\alpha, \qquad a_R = R\omega^2$$

Where a_{tan}, a_R are tangential and radial components of linear acceleration.



Linear and Rotational Quantities

Linear	Type	Rota-tional	Relation (θ in radians)
x	displacement	θ	$x = R\theta$
v	velocity	ω	$v = R\omega$
$a_{\tan}$	acceleration	α	$a_{\tan} = R\alpha$

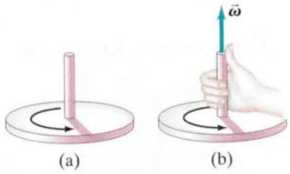


FIGURE 9 (a) Rotating wheel. (b) Right-hand rule for obtaining direction of $\vec{\omega}$.

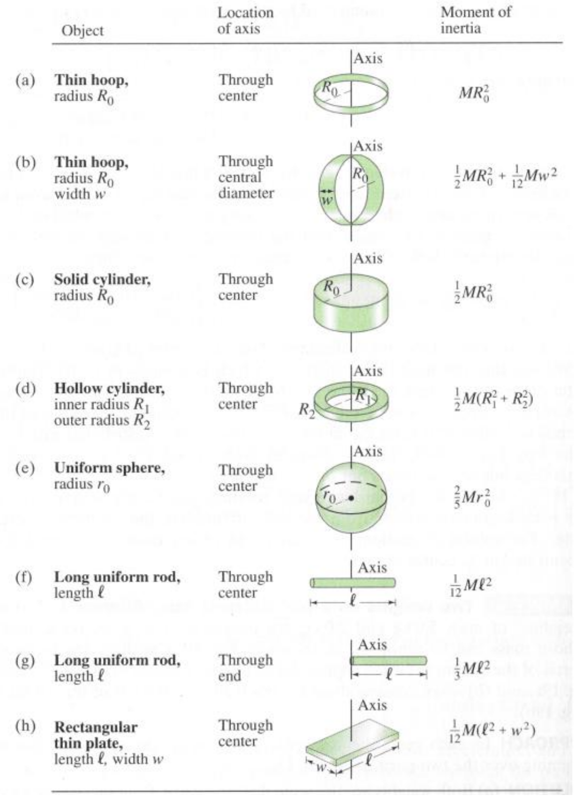
Torque:

$$\vec{\tau} = \vec{R} \times \vec{F}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Sigma\tau = I\alpha, \qquad I = \Sigma m_i R_i^2$$

I is the moment of inertia, kinda like rotational mass.



	Object	Location of axis	Moment of inertia
(a)	**Thin hoop,** radius R_0	Through center	MR_0^2
(b)	**Thin hoop,** radius R_0 width w	Through central diameter	$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder,** radius R_0	Through center	$\frac{1}{2}MR_0^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius r_0	Through center	$\frac{2}{5}Mr_0^2$
(f)	**Long uniform rod,** length ℓ	Through center	$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod,** length ℓ	Through end	$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate,** length ℓ, width w	Through center	$\frac{1}{12}M(\ell^2 + w^2)$

$$I = I_{CM} + Mh^2$$

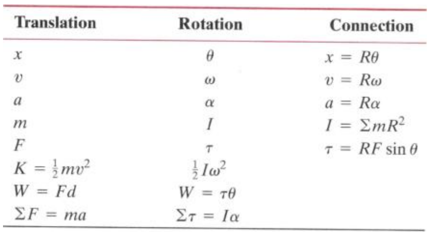
Where I_{CM} is the moment of inertia with rotation axis on the center of mass, and h is the distance from the CM to the actual axis.

$$I_z = I_x + I_y$$

$$E_{krot} = \frac{1}{2}I\omega^2$$

For shit like wheels (moving linearly and radially):

$$E_{ktot} = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$$



Translation	Rotation	Connection
x	θ	$x = R\theta$
v	ω	$v = R\omega$
a	α	$a = R\alpha$
m	I	$I = \Sigma mR^2$
F	τ	$\tau = RF\sin\theta$
$K = \frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	
$W = Fd$	$W = \tau\theta$	
$\Sigma F = ma$	$\Sigma\tau = I\alpha$	

Pendulum

$$\tau = -mgh\sin\theta$$

$$\therefore I\frac{d^2\theta}{dt^2} = -mgh\sin\theta$$

$$I\frac{d^2\theta}{dt^2} \approx -mgh\theta$$

$$I\frac{d^2\theta}{dt^2} + mgh\theta = 0$$

$$\theta \approx \theta_{max}\cos(\omega t + \phi)\,, \qquad T = 2\pi\sqrt{\frac{I}{mgh}}$$

Angular Momentum

$$L = I\omega$$

Vector relation $\vec{L} = I\vec{\omega}$ only holds around an axis of symmetry.

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Sigma\vec{\tau} = \frac{d\vec{L}}{dt}$$

Drag Force

$$F_D = -bv, \qquad F_D \propto v^2$$

(Depending on magnitude of v)

Linear Momentum

$$\vec{p} = m\vec{v}$$

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$$

Equilibrium

For equilibrium:

$$\Sigma\vec{F} = \Sigma\vec{\tau} = 0$$

- An object in static equilibrium is said to be in (a) stable, (b) unstable, or (c) neutral equilibrium, depending on whether a slight displacement leads to (a) a return to the original position, (b) further movement away from the original position, or (c) rest in a new position.
- Stress (tension, compression or shear) is the force per unit area acting on an object.
- Strain is the resulting fractional change in length.
- Elastic modulus is the ratio of stress to strain.
    - Young's modulus applies for compression and tension

    - Shear modulus applies for shear
    - Bulk modulus applies to an object whose volume changes because of pressure on all sides
    - All measured in Nm^{-2}
- Fracture is when stress breaks a material. Can be measured with:
    - Tensile strength
    - Compressive
    - Shear Strength
    - All measured in Nm^{-2}

Fluids

$$\rho = \frac{m}{V}$$

Specific gravity is the ratio of the density of the material to the density of water (at 4°C).

$$P = \frac{F}{A}$$

Stevin's Law:

$$P = \rho g h$$

$$\frac{dP}{dy} = -\rho(y)g$$

$$\therefore P(y) = P_0 e^{-\frac{y}{L}}, \qquad L = \frac{RT}{M_g} \approx 8000m$$

If $\rho \propto P$,

$$\rho(y) = \rho_0 e^{-\frac{y}{L}}$$

If density of the fluid ρ depends on y.

$$1atm = 1.013 \times 10^5 Nm^{-2}$$

$$1bar = 1 \times 10^5 Nm^{-2}$$

Pascal's principle states that if an external pressure is applied to a confined fluid, the pressure at any point within the fluid increases by that amount.

$$\frac{F_{out}}{F_{in}} = \frac{A_{out}}{A_{in}}, \qquad P_{out} = P_{in}$$

Archimedes' principle states that an object submerged wholly or partially in a fluid is buoyed up by a force equal to the weight of the fluid it displaces.

Fluid in motion can be streamline (laminar flow) or turbulent.

The equation of continuity states that for an incompressible fluid flowing in an enclosed tube, the product of the velocity of flow and the cross-sectional area of the tube remain constant:

$$Av = \text{const}$$

Bernoulli's principle states that where the velocity of the fluid is high, the pressure in it is low, and vice versa. For steady laminar flow of an incompressible and non-viscous fluid, Bernoulli's equation, which is based on the law of conservation of energy, is:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y^2$$

For pressure difference on wings during laminar flow:

$$\Delta p = \frac{1}{2}\rho\left(\mathrm{v}_{\mathrm{top}}^2 - v_{bot}^2\right)$$

$$F = \Delta P A$$

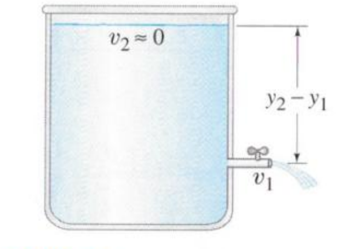


FIGURE 26 Torricelli's theorem: $v_1 = \sqrt{2g(y_2 - y_1)}$.

$$A_c \frac{dh}{dt} = A_h v$$

Viscosity (η) is measured in Pas

Oscillations

SHM

$$x(t) = Acos(\omega t + \phi)$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

For circular motion seen by profile:

$$v = v_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$$

Simple Pendulum

$$\omega = \sqrt{\frac{g}{l}}$$

$$\theta(t) = \theta_m \cos(\omega t + \phi)$$

Damped Harmonic Motion

Underdamped:

$$x(t) = Ae^{\gamma t}\cos(\omega' t + \phi)$$

$$x = e^{-\frac{ct}{2m}}\left(x_0 cos\omega t + \left(\frac{cx_0 + 2mv_0}{2\omega m}\right) sin\omega t\right)$$

$$\omega = \frac{\sqrt{4mk - c^2}}{2m}$$

Overdamped:

$$x = \left(x_0 - \frac{v_0 - x_0 r_1}{r_2 - r_1}\right)e^{r_1 t} + \left(\frac{v_0 - x_0 r_1}{r_2 - r_1}\right)e^{r_2 t}$$

$$r_1 = \frac{-c + \sqrt{c^2 - 4mk}}{2m}$$

$$r_2 = \frac{-c - \sqrt{c^2 - 4mk}}{2m}$$

Critically damped:

$$x = x_0 + \left(v_0 + \frac{cx_0}{2m}\right)te^{-\frac{c}{2m}t}$$

Forced Oscillations

If an oscillating force is applied to a system capable of vibrating, the amplitude of the vibration can be very large if the frequency of the applied force is near the natural (or resonant) frequency of the oscillator; this is called resonance.

Thermodynamics

The Zeroth Law of Thermodynamics

Two objects are said to be in thermal equilibrium if, when placed in thermal contact, no net energy flows from one to the other, and their temperatures do not change.

If two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.

The First Law of Thermodynamics

The first law of thermodynamics states that the change in internal energy of a closed system is equal to the heat added to the system, Q, minus the work, W, done by the system:

$$\Delta E_{int} = Q - W$$

The Second Law of Thermodynamics

The second law of thermodynamics concerns irreversibility. It can be stated in a variety of equivalent ways.

The Clausius version of the second law is:
Heat can flow spontaneously from a hot object to a cold object; heat will not flow spontaneously from a cold object to a hot object.

The Kelvi-Plank statement:
No device is possible whose sole effect is to transform a given amount of heat completely into work.

Natural processes tend to move towards a state of greater disorder or greater entropy.

The Third Law of Thermodynamics

As temperature approaches absolute zero, the entropy of a perfect crystal approaches zero.

The entropy change associated with any condensed system undergoing a reversible isothermal process approaches zero as the temperature at which it is performed approaches 0 K.

It is impossible for any process, no matter how idealized, to reduce the entropy of a system to its absolute-zero value in a finite number of operations.

Thermal Expansion

The change in length Δl of a solid when temperature changes by ΔT is directly proportional to ΔT and to its original length.

$$\Delta l = \alpha l_0 \Delta T$$

Where α is the coefficient of linear expansion.

$$\Delta V = \beta V_0 \Delta T$$

Where $\beta \approx 3a$ is the coefficient of the volume expansion.

$$PV = nRT$$

Kinetic Theory of Gases

$$E_K = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$

Maxwellian distribution of velocity in a gas of temperature T:

$$f_M(v) = \frac{N}{\left(\frac{2\pi k_BT}{m}\right)^{\frac{3}{2}}}e^{-\frac{mv^2}{2k_BT}}$$

$$f(v) = 4\pi v^2 f_M(v)$$

Van der Waals equation of state:

$$\left(P + \frac{a}{\left(\frac{V}{n}\right)^2}\right)\left(\frac{V}{n} - b\right) = RT$$

Below the critical temperature, a gas can change to a liquid if sufficient pressure is applied; but if the temperature is higher than the critical temperature, no amount of pressure will cause a liquid to form.

The triple point of a substance is the unique temperature and pressure at which all three phases - solid, liquid and gas - can coexist in equilibrium. Because of its precise reproducibility, the triple point of water is often taken as a standard reference point.

In physics a vapor (American English spelling) or vapour (British) is a substance in the gas phase at a temperature lower than its critical point, which means that the vapor can be condensed to a liquid by increasing the pressure on it without reducing the temperature. A vapor is different than an aerosol. An aerosol is a suspension of tiny particles of liquid, solid, or both within a gas.

For example, water has a critical temperature of 374 °C (647 K), which is the highest temperature at which liquid water can exist. In the atmosphere at ordinary temperatures, therefore, gaseous water (known as water vapor) will condense into a liquid if its partial pressure is increased sufficiently.

A vapor may co-exist with a liquid (or a solid). When this is true, the two phases will be in equilibrium, and the gas-partial pressure will be equal to the equilibrium vapor pressure of the liquid (or solid).

Vapor is responsible for the familiar processes of cloud formation and condensation.

Vapor Pressure

The vapor pressure is the equilibrium pressure from a liquid or a solid at a specific temperature. The equilibrium vapor pressure of a liquid or solid is not affected by the amount of contact with the liquid or solid interface.

The normal boiling point of a liquid is the temperature at which the vapor pressure is equal to normal atmospheric pressure.

Saturated vapor pressure refers to the pressure of the vapor above the liquid when the phases are in equilibrium. The vapor pressure of a substance (such as water) depends strongly on temperature and is equal to atmospheric pressure at boiling point.

Relative humidity of air at a given place is the ratio of the partial pressure of water vapor in the air to the saturated vapor pressure at that temperature; it is usually expressed as a percentage.

The mean free path is the average distance a molecule moves between collisions with other molecules. It is inversely proportional to the collisional cross section and to the gas number density:

$$\lambda_{mfp} = \frac{1}{\sigma \rho_N}$$

σ: **Collision cross section** (how “big” a target the particles present to each other)
ρ_N: **Number density** (number of particles per unit volume)

Diffusion is the process whereby molecules of a substance move (on average) from one area to another because of a difference in concentration. The rate of diffusion, J, is directly proportional to the concentration gradient (Fick’s law):

$$J = DA\frac{dc}{dx}$$

J: **Diffusion flux** – how much stuff flows per unit time (e.g., moles/sec)
D: **Diffusion coefficient** – how fast stuff spreads (depends on the medium and particle)
D: **Area** through which diffusion happens
$\frac{dC}{dx}$: **Concentration gradient** – how quickly concentration changes with position

$$E_{int} = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

If the gas is not monoatomic, then rotational and vibrational energies must also be considered.

$$Q = mc\Delta T$$

- A system is any object or set of objects we wish to consider; everything outside the system is the environment or surroundings.
    - Closed system: no mass enters or leaves, but energy may be exchanged with the environment.

  - Open system: mass as well as energy may be exchanged with the environment.
  - Isolated system: no mass or energy may be exchanged with the environment.

The work done by (or on) a gas to change its volume by dV is:

$$dW = PdV$$

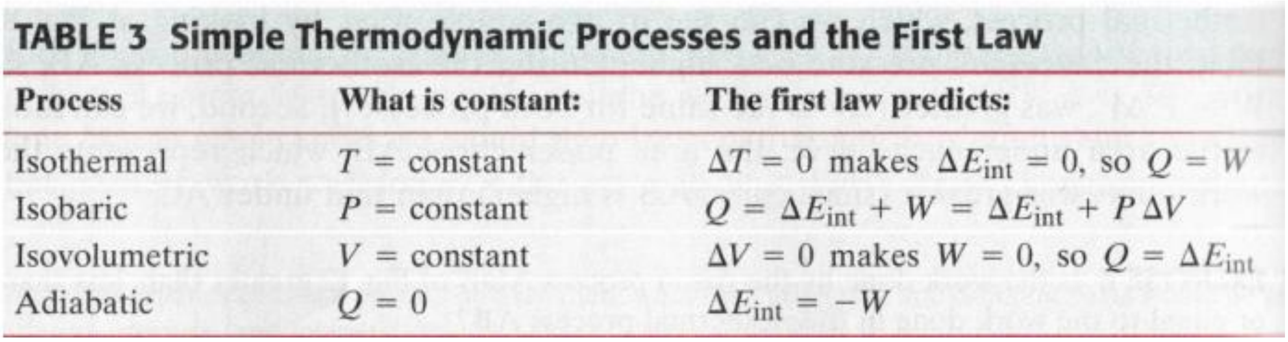


TABLE 3 Simple Thermodynamic Processes and the First Law

Process	What is constant:	The first law predicts:
Isothermal	$T = \text{constant}$	$\Delta T = 0$ makes $\Delta E_{\text{int}} = 0$, so $Q = W$
Isobaric	$P = \text{constant}$	$Q = \Delta E_{\text{int}} + W = \Delta E_{\text{int}} + P\,\Delta V$
Isovolumetric	$V = \text{constant}$	$\Delta V = 0$ makes $W = 0$, so $Q = \Delta E_{\text{int}}$
Adiabatic	$Q = 0$	$\Delta E_{\text{int}} = -W$

Work and heat are not function of the state of a system (as are P, V, T, n and E_{int}), but depend on the type of process that takes the system from one state to another.

The molar specific heat of an ideal gas at constant volume, C_V, and at constant pressure, C_P, are defined by

$$Q_V = nC_V\Delta T$$

$$Q_P = nC_P\Delta T$$

$$C_P - C_V = R$$

For monoatomic ideal gas:

$$C_V = \frac{3}{2}R$$

For ideal gases made up of diatomic or more complex molecules, C_V is equal to $\frac{R}{2}$ times the number of degrees of freedom of the molecule. Unless the temperature is very high, some of the degrees of freedom are not active and so do not contribute.

According to the principle of equipartition of energy, energy is shared equally among the active degrees of freedom in an amount $\frac{1}{2}k_BT$ per molecule on average.

If $Q = 0$ (adiabatic expansion):

$$PV^{\gamma} = PV^{\frac{C_P}{C_V}} = \text{const}$$

Conduction:

$$\frac{\Delta Q}{\Delta T} = kA\frac{(T_1 - T_2)}{l}$$

Radiation (Stefan-Boltzmann Law):

$$\frac{\Delta Q}{\Delta T} = \epsilon\sigma AT^4$$

Solar constant:

$$S_C = 1350Wm^{-2}$$

$$\frac{\Delta Q}{\Delta t} = S_c\epsilon A\cos\theta$$

Second Law of Thermodynamics

A heat engine is a device for changing thermal energy, by means of heat flow, into useful work.

$$Q_H = W + Q_L$$

$$e = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

A reversible process is one that is carried out infinitely slowly, so that the process can be considered as a sequence of equilibrium states, and friction is not involved, so that the whole process could be done in reverse with no change in magnitude of the work and/or heat exchanged.

All real processes are irreversible: they are not done infinitely slowly, so that turbulence is involved, and friction is always present. Nevertheless, a real process may approximate a reversible process and therefore reversible process are conceptually important as representing the theoretically maximum performance of an engine.

Carnot Engine

Carnot's (idealized) engine consists of two isothermal and two adiabatic processes in a reversible cycle. The efficiency of the Carnot cycle is:

$$e_{ideal} = \frac{W}{Q_H} = 1 - \frac{T_L}{T_H}$$

Actual engines always have lower efficiencies than this.

Carnot's Theorem:

All reversible engines operating between the same two constant temperatures have the same efficiency. Any irreversible engine operating between the same two fixed temperatures will have an efficiency less than this.

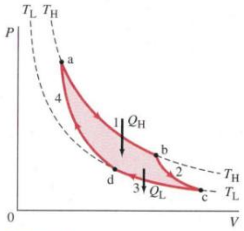
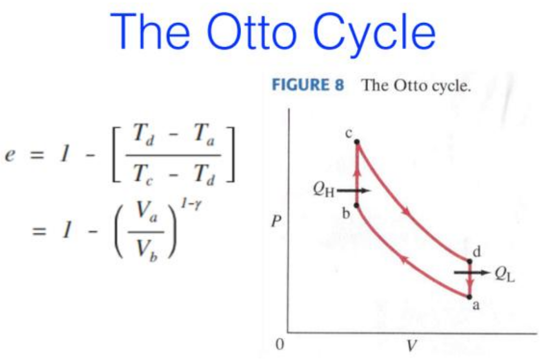


FIGURE 7 The Carnot cycle. Heat engines work in a cycle, and the cycle for the Carnot engine begins at point a on this *PV* diagram. (1) The gas is first expanded isothermally, with the addition of heat Q_H, along the path ab at temperature T_H. (2) Next the gas expands adiabatically from b to c—no heat is exchanged, but the temperature drops to T_L. (3) The gas is then compressed at constant temperature T_L, path cd, and heat Q_L flows out. (4) Finally, the gas is compressed adiabatically, path da, back to its original state. No Carnot engine actually exists, but as a theoretical idea it played an important role in the development of thermodynamics.

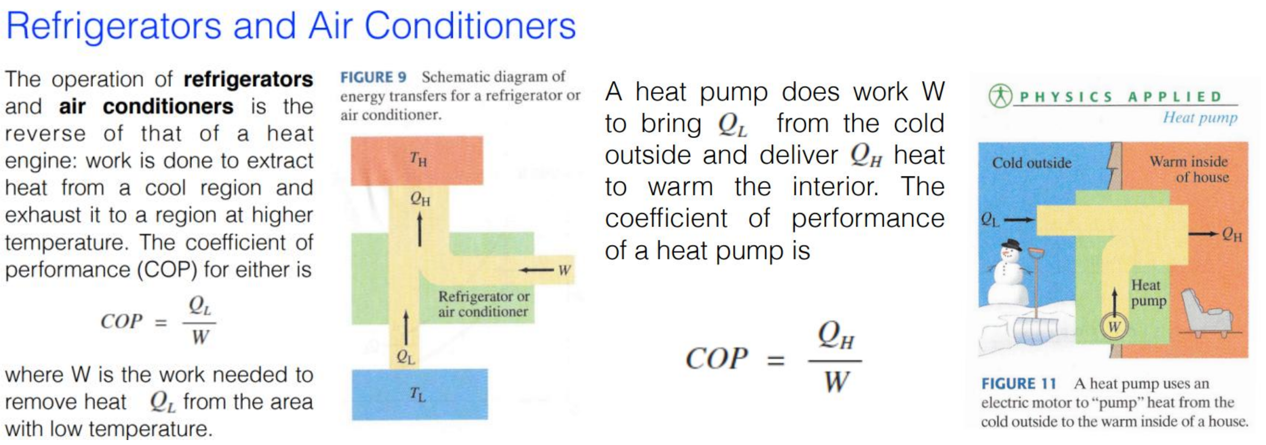


The Otto Cycle

$$e = 1 - \left[\frac{T_d - T_a}{T_c - T_d} \right]$$

$$= 1 - \left(\frac{V_a}{V_b} \right)^{1-\gamma}$$

FIGURE 8 The Otto cycle.



Refrigerators and Air Conditioners

The operation of **refrigerators** and **air conditioners** is the reverse of that of a heat engine: work is done to extract heat from a cool region and exhaust it to a region at higher temperature. The coefficient of performance (COP) for either is

$$COP = \frac{Q_L}{W}$$

where W is the work needed to remove heat Q_L from the area with low temperature.

FIGURE 9 Schematic diagram of energy transfers for a refrigerator or air conditioner.

A heat pump does work W to bring Q_L from the cold outside and deliver Q_H heat to warm the interior. The coefficient of performance of a heat pump is

$$COP = \frac{Q_H}{W}$$

FIGURE 11 A heat pump uses an electric motor to "pump" heat from the cold outside to the warm inside of a house.

$$COP_{ideal} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L}$$

Entropy

$$\Delta S = \frac{Q}{T}$$

$$dS = \frac{dQ}{T}$$

For cooling:

$$\Delta S = mc \ln \frac{T_f}{T_i}$$

For any reversible cycle:

$$\Delta S = 0$$

Otherwise, overall

$$\Delta S > 0$$

Statistical definition of entropy:

$$S = k_B \ln W$$

Where W is the thermodynamic probability or disorder parameter.

The third law of thermodynamics is sometimes stated as follows, regarding the properties of systems in equilibrium at absolute zero temperature:
The entropy of a perfect crystal at absolute zero is exactly equal to zero.

Entropy is related to the number of accessible microstates, and for a system consisting of many particles, quantum mechanics indicates that there is only one unique microstate (called the ground state) with minimum energy. Then,

$$W = 1 \Rightarrow \ln W = 0 \Rightarrow S = 0$$

The Nernst–Simon statement of the third law of thermodynamics concerns thermodynamic processes at a fixed, low temperature: The entropy change associated with any condensed system undergoing a reversible isothermal process approaches zero as the temperature at which it is performed approaches 0 K. A classical formulation by Nernst (a consequence of the Third Law) is: It is impossible for any process, no matter how idealized, to reduce the entropy of a system to its absolute-zero value in a finite number of operations. Physically, the Nernst–Simon statement implies that it is impossible for any procedure to bring a system to the absolute zero of temperature in a finite number of steps.

Constants (A-Z)

$$1cal = 4.186J$$

$$G = 6.67 \times 10^{-11}Nm^{2}Kg^{-1}$$

$$k_{B} = \frac{R}{N_{A}} = 1.38 \times 10^{-23}JK^{-1}$$

$$M_{E} = 5.98 \times 10^{24}Kg$$

$$N_{A} = 6.02 \times 10^{23}$$

$$R = 8.314Jmol^{-1}K^{-1}$$

$$r_{E} = 6380km$$

$$\sigma = 5.67 \times 10^{-8}Wm^{-2}K^{-4}$$

$$S_{C} = 1350Wm^{-2}$$

$$1u = 1.6605 \times 10^{-27}kg$$

$$0°C = 273.15K$$

Analysis Notes

Integration By Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

Solving Separable Variable Differential Equations

$$y' = h(x)g(y)$$

$$\frac{y'}{g(y)} = h(x)$$

Integrate both sides with respect to x.

Solving Homogeneous (in $\boldsymbol{x}, \boldsymbol{y}$) Differential Equations

$$y' = f\left(\frac{y}{x}\right)$$

$$y' = f(z), z = \frac{y}{x}$$

$$\therefore y = zx$$

$$y' = z'x + z$$

$$f(z) = z'x + z$$

$$z' = \frac{f(z) - z}{x}$$

Separable variable equation

Solving 1st Order Homogeneous ODEs

$$y' + a(x)y = 0$$

$$y' = ya(x)$$

Separable

Solving 1st Order Non-Homogeneous/Complete ODEs

$$y' + a(x)y = b(x)$$

$$y = f(x) + g(x) : f'(x) + a(x)f(x) = 0$$

Lagrange's Variation of Parameters

$$g(x) = k(x)e^{-A(x)} : A'(x) = a(x)$$

$$g'(x) + a(x)g(x) = b(x)$$

$$\left(k(x)e^{-A(x)}\right)' + a(x)\left(k(x)e^{-A(x)}\right) = b(x)$$

$$k'(x)e^{-A(x)} - k(x)a(x)e^{-A(x)} + a(x)k(x)e^{-A(x)} = b(x)$$

$$k'(x)e^{-A(x)} = b(x)$$

$$k(x) = \int b(x)e^{A(x)}dx$$

$$g(x) = \int b(x)e^{A(x)}dx \cdot e^{-A(x)}$$

$$\therefore y = \int b(x)e^{A(x)}dx \cdot e^{-A(x)} + f(x)\colon f'(x) + a(x)f(x) = 0$$

Solving Linear ODEs of II Order

$$y'' + a(x)y' + b(x)y = p(x) \; a, b, p \in C^0(I)$$

Solution is sum of:

1. General integral of the associated homogeneous equation
2. A particular integral of the complete equation

Solving Associated Homogeneous Equation

$$y'' + a(x)y' + b(x)y = 0$$

Solution is in the form

$$c_1\varphi_1(x) + c_2\varphi_2(x) \; c_1, c_2 \in \mathbb{R}, \exists x \in I : \varphi_1(x) \neq k\varphi_2(x)$$

$$y'' + ay' + by = 0 \; a, b \in \mathbb{R}$$

$$y = e^{\lambda x}$$

$$y' = \lambda e^{\lambda x}$$

$$y'' = \lambda^2 e^{\lambda x}$$

$$e^{\lambda x}(\lambda^2 + a\lambda + b) = 0$$

$$\lambda^2 + a\lambda + b = 0$$

Solve quadratic (characteristic equation)

$$a^2 - 4b = 0$$

$$\Rightarrow y = (c_1 + c_2 x)e^{\lambda x}$$

Solving Particular Solution of the Complete Equation

1. Lagrange's method
2. Similarity method

$$p(x) = p_n(x)e^{\mu x}$$

Where $p_n(x)$ is a polynomial of degree n and $\mu \in \mathbb{C}$.

$$\bar{y} = q_n(x)e^{\mu x}, \mu \neq \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$\bar{y} = x q_n(x)e^{\mu x}, \mu = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$\bar{y} = x^2 q_n(x)e^{\mu x}, \mu = \frac{-a + \sqrt{a^2 - 4b}}{2} = \frac{-a - \sqrt{a^2 - 4b}}{2}$$

Case $\boldsymbol{p(x)}$ is a Trigonometric Function

e.g.

$$y'' + y = tcost$$

$$y'' + y = \frac{t}{2}e^{it} + \frac{t}{2}e^{-it}$$

Solution of equation is:

$$y = y_1 + y_2$$

$$y_1'' + y_1 = \frac{t}{2}e^{it}$$

$$y_2'' + y_2 = \frac{t}{2}e^{-it}$$

Cauchy Problem Details

$$y' = f(y, x)$$

$$y(x_0) = y_0, x \in I$$

$$y \ continuous$$

$$\Rightarrow y \ admits \ at \ least \ one \ local \ (to \ I) \ solution$$

In addition, to verify if the solution is unique, the following condition (Lipschitz condition) must be met:

$$y'(x) = l \in \mathbb{R}\ \forall x \in I$$

Tables

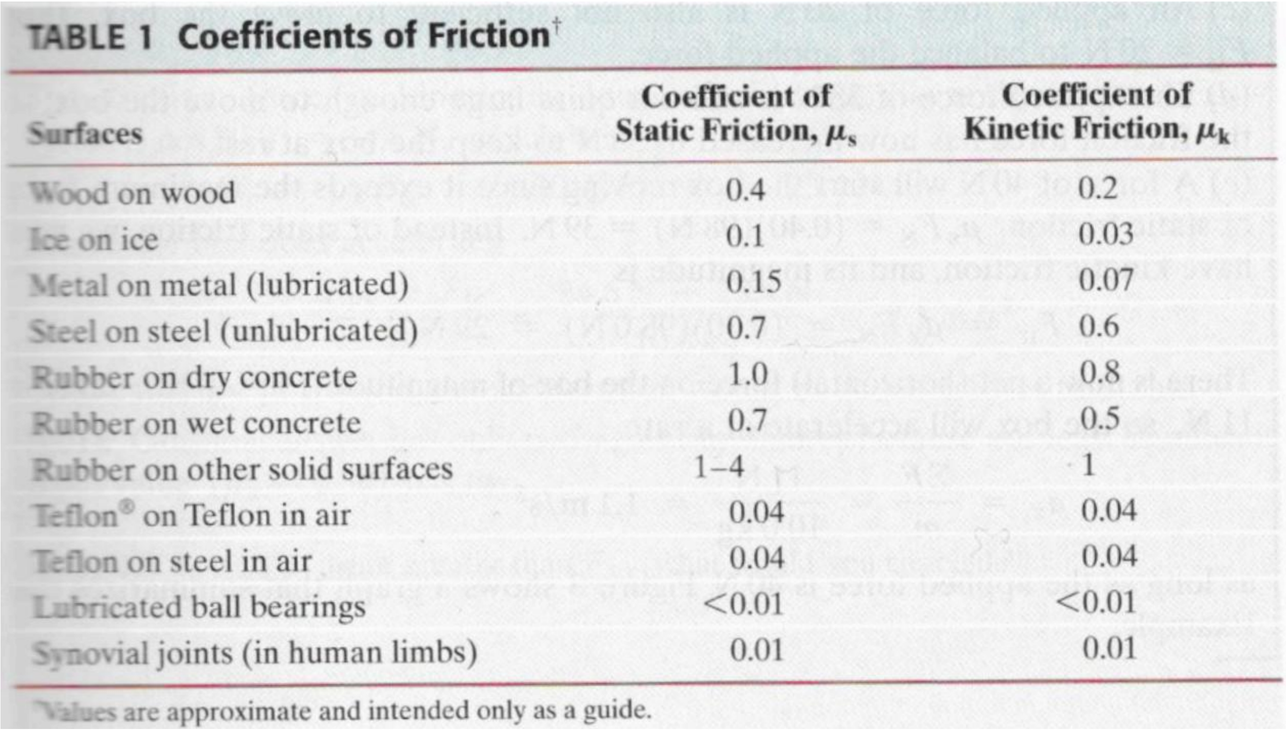


TABLE 1 Coefficients of Friction†

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

† Values are approximate and intended only as a guide.

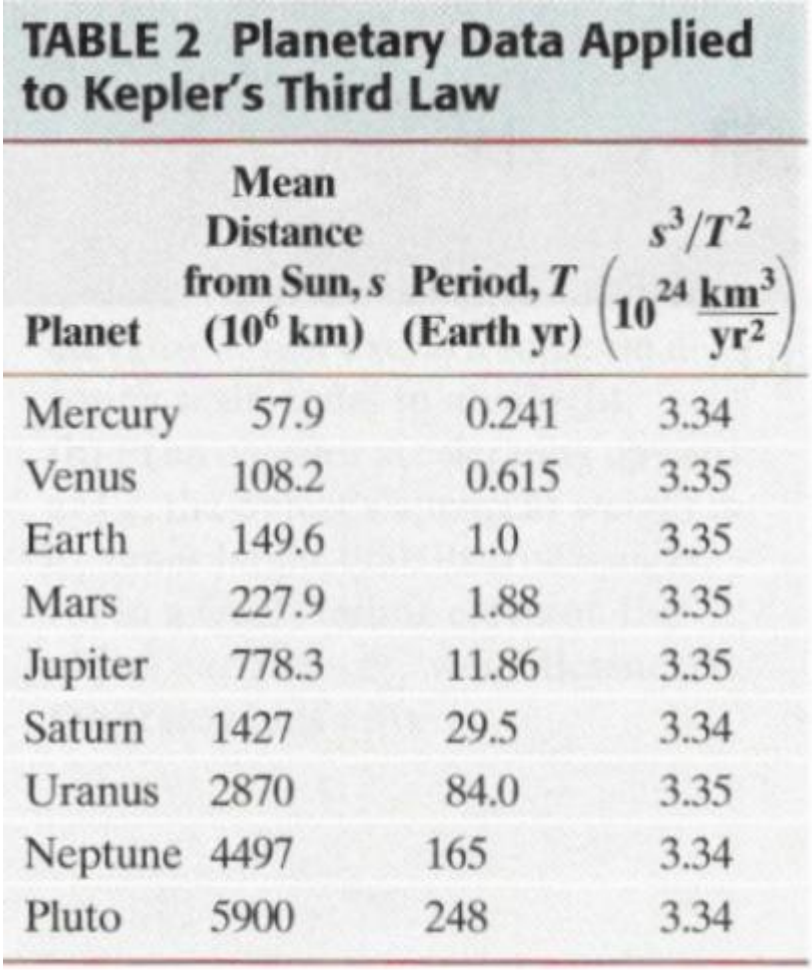


TABLE 2 Planetary Data Applied to Kepler's Third Law

Planet	Mean Distance from Sun, s (10^6 km)	Period, T (Earth yr)	s^3/T^2 $\left(10^{24}\,\frac{\text{km}^3}{\text{yr}^2}\right)$
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.0	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
Pluto	5900	248	3.34

TABLE 1 Elastic Moduli

Material	Young's Modulus, E (N/m^2)	Shear Modulus, G (N/m^2)	Bulk Modulus, B (N/m^2)
Solids			
Iron, cast	100×10^9	40×10^9	90×10^9
Steel	200×10^9	80×10^9	140×10^9
Brass	100×10^9	35×10^9	80×10^9
Aluminum	70×10^9	25×10^9	70×10^9
Concrete	20×10^9		
Brick	14×10^9		
Marble	50×10^9		70×10^9
Granite	45×10^9		45×10^9
Wood (pine) (parallel to grain)	10×10^9		
(perpendicular to grain)	1×10^9		
Nylon	5×10^9		
Bone (limb)	15×10^9	80×10^9	
Liquids			
Water			2.0×10^9
Alcohol (ethyl)			1.0×10^9
Mercury			2.5×10^9
Gases[†]			
Air, H_2, He, CO_2			1.01×10^5

[†] At normal atmospheric pressure; no variation in temperature during process.

TABLE 2 Ultimate Strengths of Materials (force/area)

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)	Shear Strength (N/m^2)
Iron, cast	170×10^6	550×10^6	170×10^6
Steel	500×10^6	500×10^6	250×10^6
Brass	250×10^6	250×10^6	200×10^6
Aluminum	200×10^6	200×10^6	200×10^6
Concrete	2×10^6	20×10^6	2×10^6
Brick		35×10^6	
Marble		80×10^6	
Granite		170×10^6	
Wood (pine) (parallel to grain)	40×10^6	35×10^6	5×10^6
(perpendicular to grain)		10×10^6	
Nylon	500×10^6		
Bone (limb)	130×10^6	170×10^6	

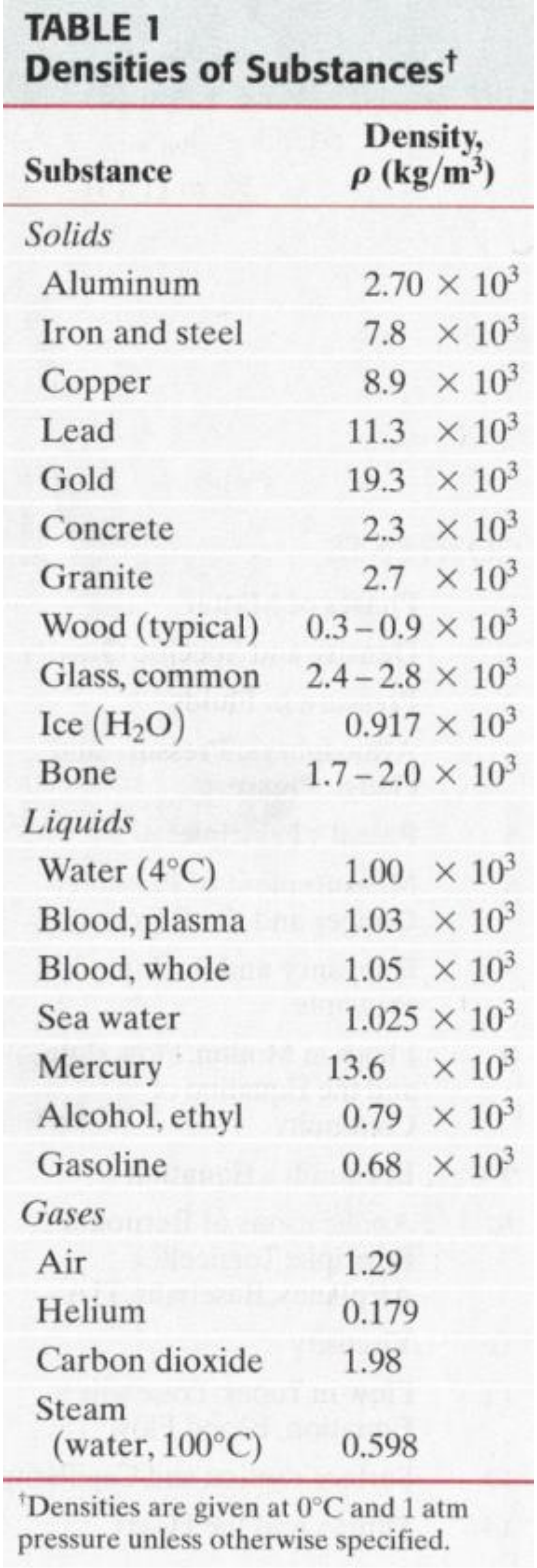


TABLE 1

Densities of Substances[†]

Substance	Density, ρ (kg/m^3)
Solids	
Aluminum	2.70×10^3
Iron and steel	7.8×10^3
Copper	8.9×10^3
Lead	11.3×10^3
Gold	19.3×10^3
Concrete	2.3×10^3
Granite	2.7×10^3
Wood (typical)	$0.3-0.9 \times 10^3$
Glass, common	$2.4-2.8 \times 10^3$
Ice (H_2O)	0.917×10^3
Bone	$1.7-2.0 \times 10^3$
Liquids	
Water (4°C)	1.00×10^3
Blood, plasma	1.03×10^3
Blood, whole	1.05×10^3
Sea water	1.025×10^3
Mercury	13.6×10^3
Alcohol, ethyl	0.79×10^3
Gasoline	0.68×10^3
Gases	
Air	1.29
Helium	0.179
Carbon dioxide	1.98
Steam (water, 100°C)	0.598

[†] Densities are given at 0°C and 1 atm pressure unless otherwise specified.

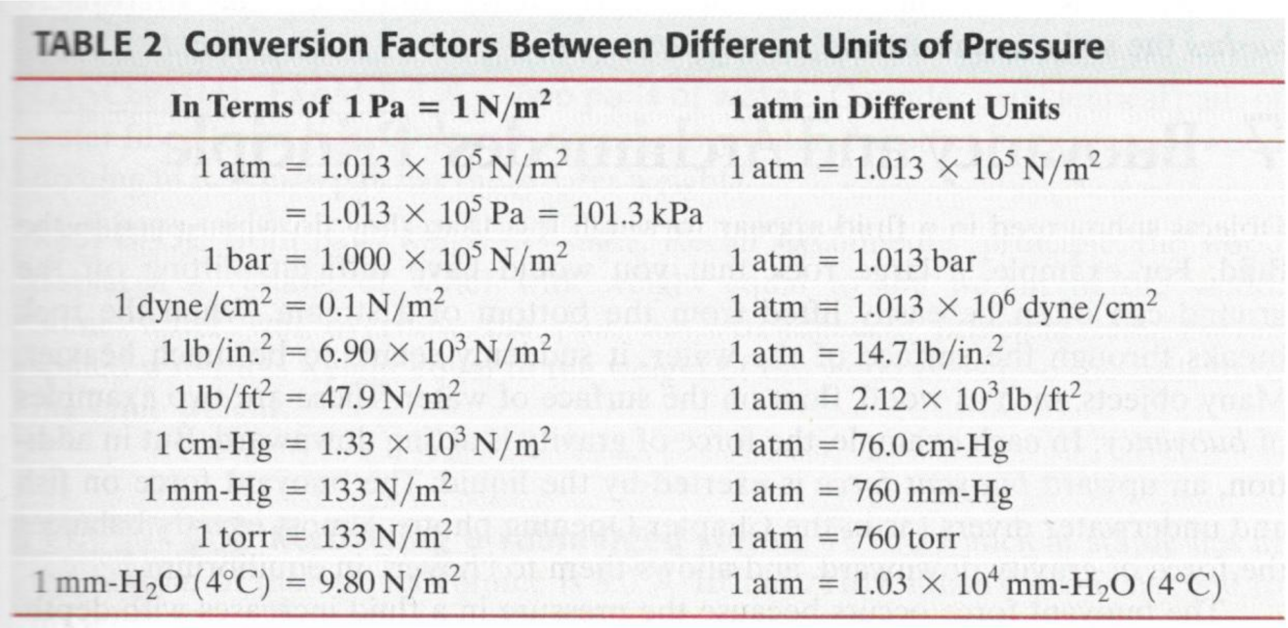


TABLE 2 Conversion Factors Between Different Units of Pressure

In Terms of $1\ \text{Pa} = 1\ \text{N/m}^2$	1 atm in Different Units
$1\ \text{atm} = 1.013 \times 10^5\ \text{N/m}^2$	$1\ \text{atm} = 1.013 \times 10^5\ \text{N/m}^2$
$= 1.013 \times 10^5\ \text{Pa} = 101.3\ \text{kPa}$	
$1\ \text{bar} = 1.000 \times 10^5\ \text{N/m}^2$	$1\ \text{atm} = 1.013\ \text{bar}$
$1\ \text{dyne/cm}^2 = 0.1\ \text{N/m}^2$	$1\ \text{atm} = 1.013 \times 10^6\ \text{dyne/cm}^2$
$1\ \text{lb/in.}^2 = 6.90 \times 10^3\ \text{N/m}^2$	$1\ \text{atm} = 14.7\ \text{lb/in.}^2$
$1\ \text{lb/ft}^2 = 47.9\ \text{N/m}^2$	$1\ \text{atm} = 2.12 \times 10^3\ \text{lb/ft}^2$
$1\ \text{cm-Hg} = 1.33 \times 10^3\ \text{N/m}^2$	$1\ \text{atm} = 76.0\ \text{cm-Hg}$
$1\ \text{mm-Hg} = 133\ \text{N/m}^2$	$1\ \text{atm} = 760\ \text{mm-Hg}$
$1\ \text{torr} = 133\ \text{N/m}^2$	$1\ \text{atm} = 760\ \text{torr}$
$1\ \text{mm-H}_2\text{O}\ (4°\text{C}) = 9.80\ \text{N/m}^2$	$1\ \text{atm} = 1.03 \times 10^4\ \text{mm-H}_2\text{O}\ (4°\text{C})$

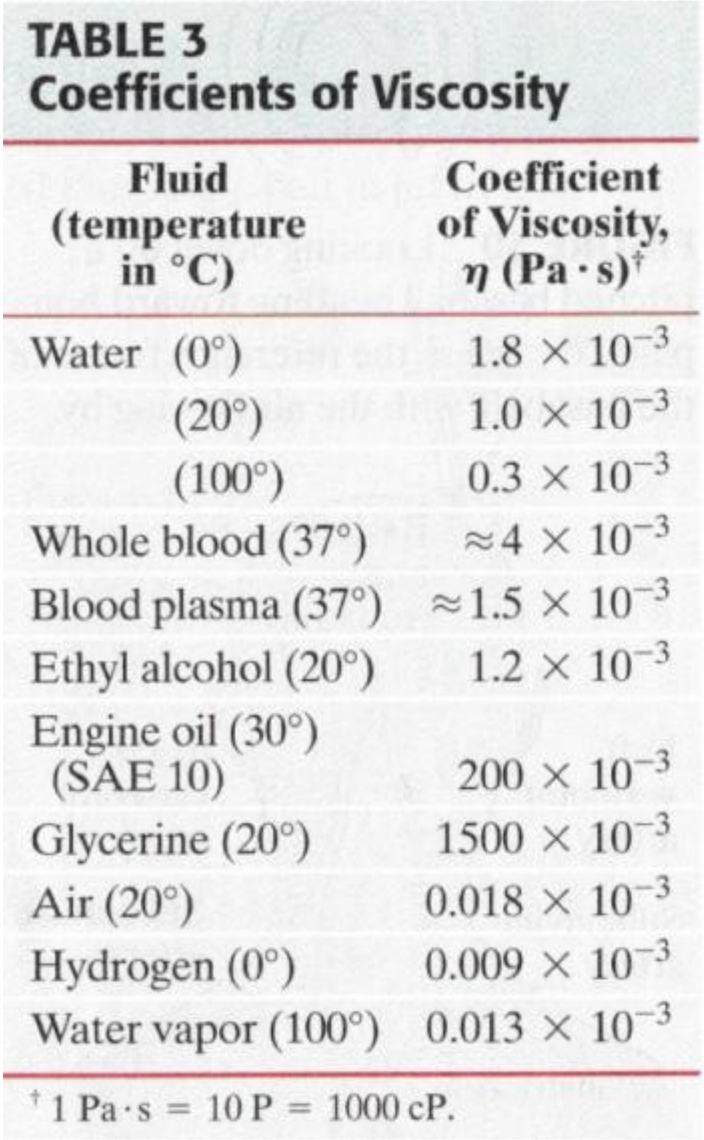


TABLE 3
Coefficients of Viscosity

Fluid (temperature in °C)	Coefficient of Viscosity, η (Pa · s)[†]
Water (0°)	1.8×10^{-3}
(20°)	1.0×10^{-3}
(100°)	0.3×10^{-3}
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	1.2×10^{-3}
Engine oil (30°) (SAE 10)	200×10^{-3}
Glycerine (20°)	1500×10^{-3}
Air (20°)	0.018×10^{-3}
Hydrogen (0°)	0.009×10^{-3}
Water vapor (100°)	0.013×10^{-3}

[†] 1 Pa · s = 10 P = 1000 cP.

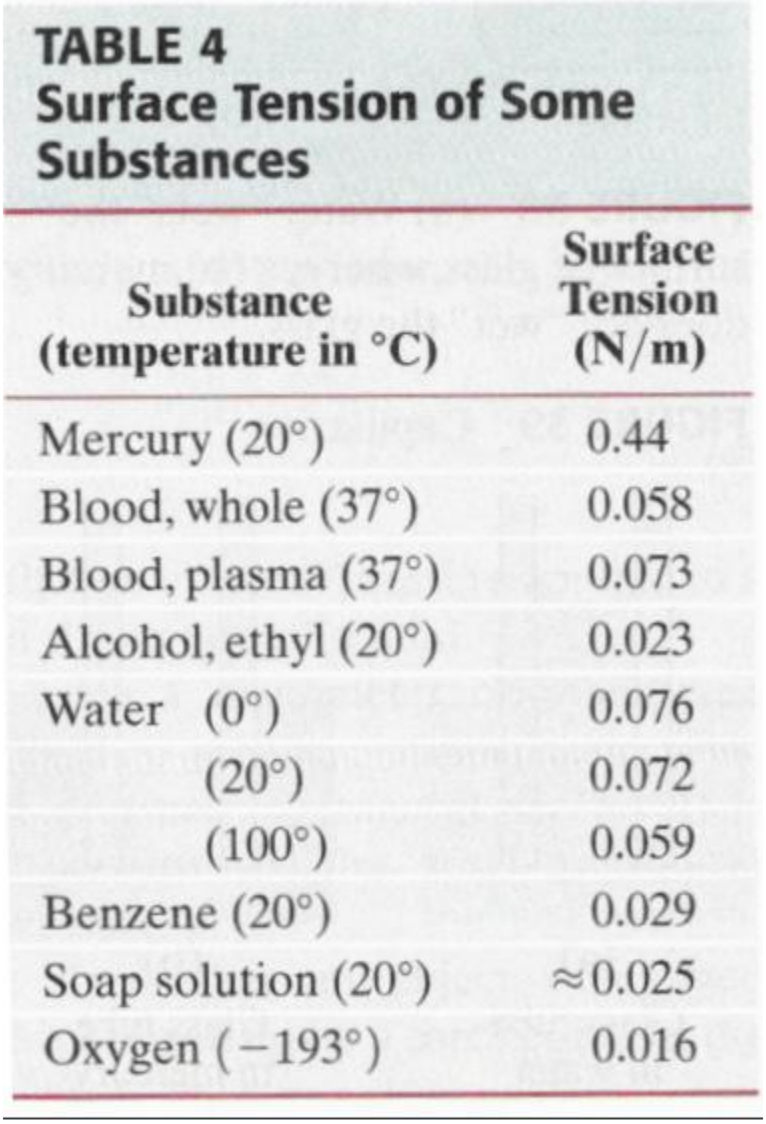


TABLE 4
Surface Tension of Some Substances

Substance (temperature in °C)	Surface Tension (N/m)
Mercury (20°)	0.44
Blood, whole (37°)	0.058
Blood, plasma (37°)	0.073
Alcohol, ethyl (20°)	0.023
Water (0°)	0.076
(20°)	0.072
(100°)	0.059
Benzene (20°)	0.029
Soap solution (20°)	≈0.025
Oxygen (−193°)	0.016

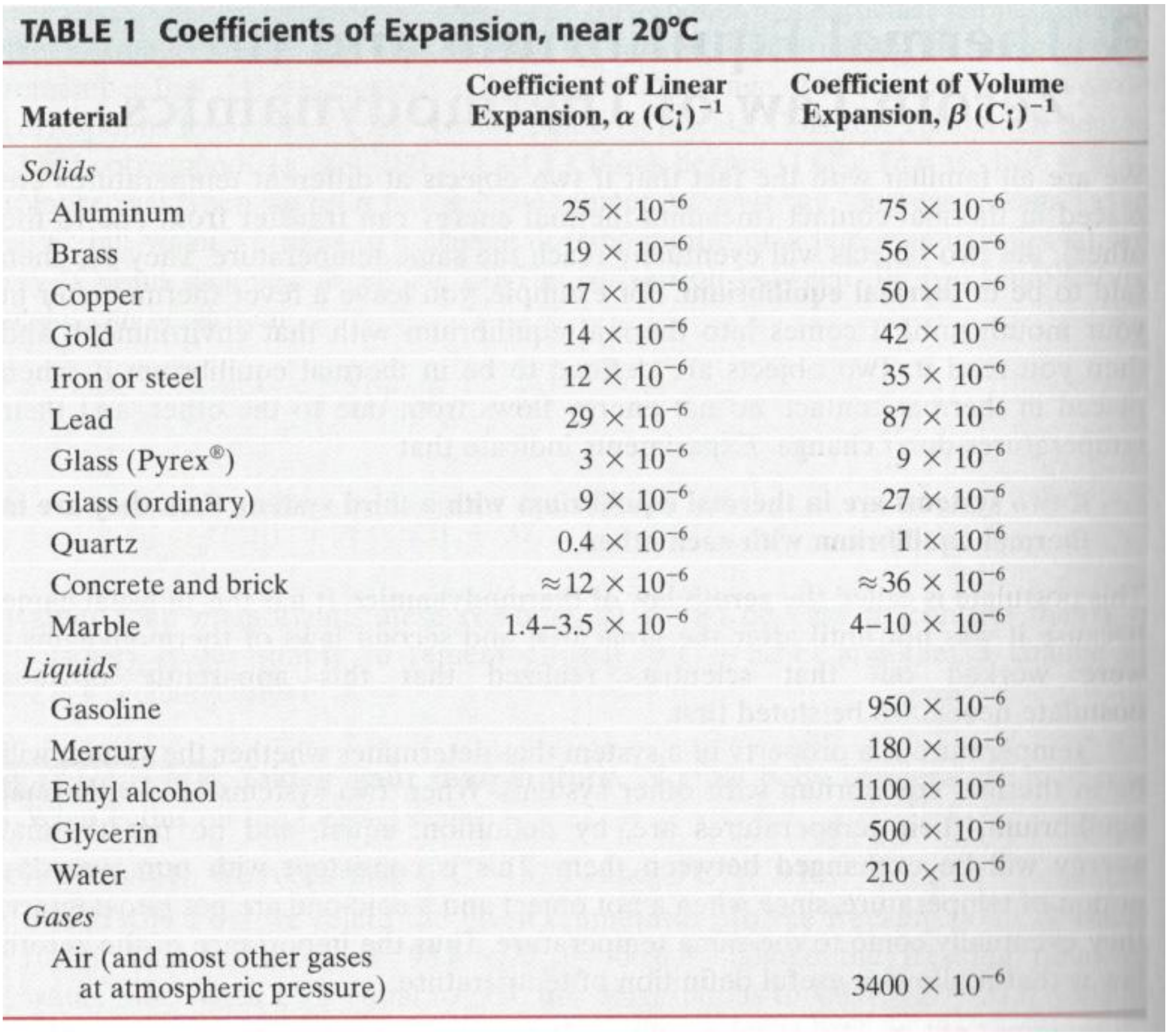


TABLE 1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α (C¡)$^{-1}$	Coefficient of Volume Expansion, β (C¡)$^{-1}$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	50×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or steel	12×10^{-6}	35×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$	$\approx 36 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$	$4\text{–}10 \times 10^{-6}$
Liquids		
Gasoline		950×10^{-6}
Mercury		180×10^{-6}
Ethyl alcohol		1100×10^{-6}
Glycerin		500×10^{-6}
Water		210×10^{-6}
Gases		
Air (and most other gases at atmospheric pressure)		3400×10^{-6}

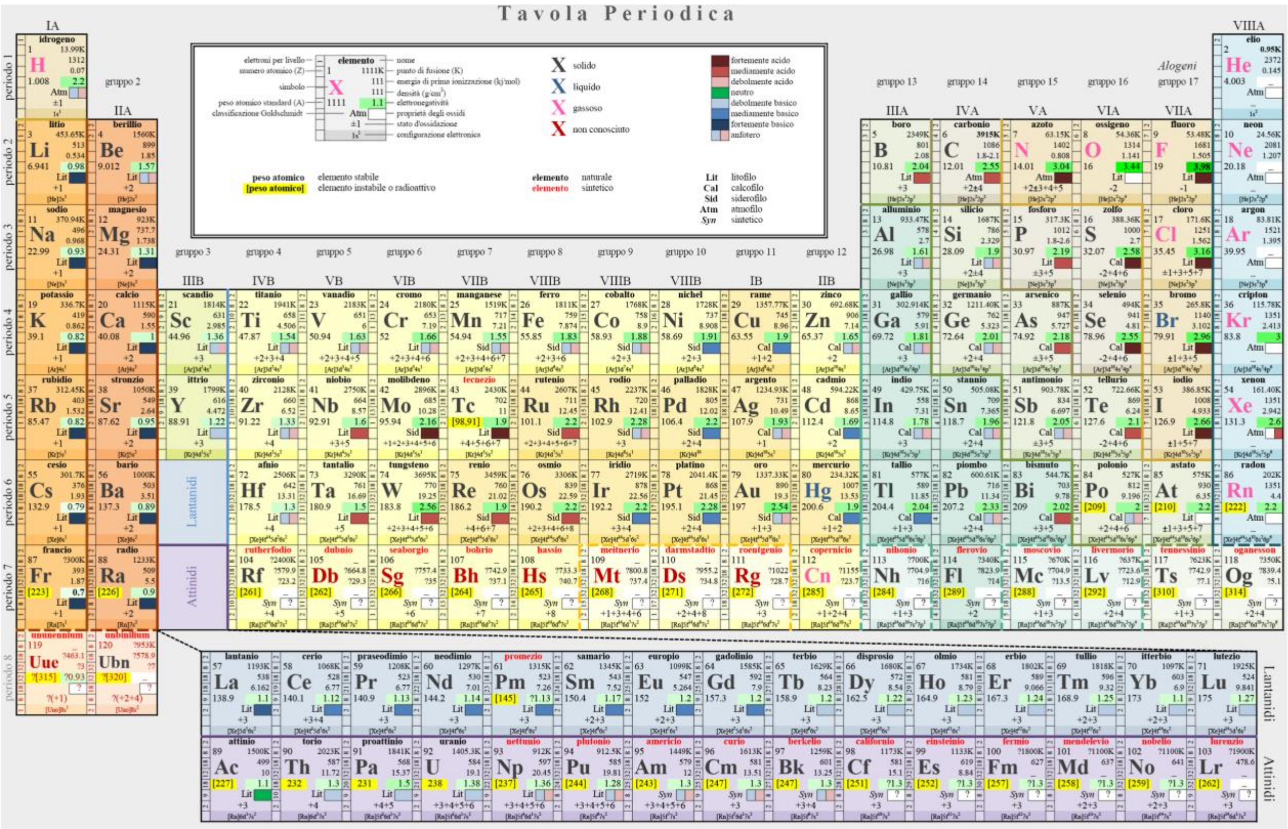


Tavola Periodica
elettroni per livello
numero atomico (Z)
simbolo
peso atomico standard (A)
classificazione Goldschmidt
elemento
nome
punto di fusione (K)
energia di prima ionizzazione (kj/mol)
densità (g/cm³)
elettronegatività
proprietà degli ossidi
stato d'ossidazione
configurazione elettronica
X solido
X liquido
X gassoso
X non conosciuto
fortemente acido
mediamente acido
debolmente acido
neutro
debolmente basico
mediamente basico
fortemente basico
anfotero
peso atomico elemento stabile
[peso atomico] elemento instabile o radioattivo
elemento naturale
elemento sintetico
Lit litofilo
Cal calcofilo
Sid siderofilo
Atm atmofilo
Syn sintetico
IA
IIA
IIIB
IVB
VB
VIB
VIIB
VIIIB
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VIIIB
IB
IIB
IIIA
IVA
VA
VIA
VIIA
VIIIA
gruppo 2
gruppo 3
gruppo 4
gruppo 5
gruppo 6
gruppo 7
gruppo 8
gruppo 9
gruppo 10
gruppo 11
gruppo 12
gruppo 13
gruppo 14
gruppo 15
gruppo 16
Alogeni
gruppo 17
periodo 1
periodo 2
periodo 3
periodo 4
periodo 5
periodo 6
periodo 7
periodo 8
Lantanidi
Attinidi
idrogeno H
elio He
litio Li
berillio Be
boro B
carbonio C
azoto N
ossigeno O
fluoro F
neon Ne
sodio Na
magnesio Mg
alluminio Al
silicio Si
fosforo P
zolfo S
cloro Cl
argon Ar
potassio K
calcio Ca
scandio Sc
titanio Ti
vanadio V
cromo Cr
manganese Mn
ferro Fe
cobalto Co
nichel Ni
rame Cu
zinco Zn
gallio Ga
germanio Ge
arsenico As
selenio Se
bromo Br
cripton Kr
rubidio Rb
stronzio Sr
ittrio Y
zirconio Zr
niobio Nb
molibdeno Mo
tecnezio Tc
rutenio Ru
rodio Rh
palladio Pd
argento Ag
cadmio Cd
indio In
stannio Sn
antimonio Sb
tellurio Te
iodio I
xenon Xe
cesio Cs
bario Ba
afnio Hf
tantalio Ta
tungsteno W
renio Re
osmio Os
iridio Ir
platino Pt
oro Au
mercurio Hg
tallio Tl
piombo Pb
bismuto Bi
polonio Po
astato At
radon Rn
francio Fr
radio Ra
rutherfodio Rf
dubnio Db
seaborgio Sg
bohrio Bh
hassio Hs
meitnerio Mt
darmstadtio Ds
roentgenio Rg
copernicio Cn
nihonio Nh
flerovio Fl
moscovio Mc
livermorio Lv
tennessinio Ts
oganesson Og
ununennium Uue
unbinilium Ubn
lantanio La
cerio Ce
praseodimio Pr
neodimio Nd
promezio Pm
samario Sm
europio Eu
gadolinio Gd
terbio Tb
disprosio Dy
olmio Ho
erbio Er
tullio Tm
itterbio Yb
lutezio Lu
attinio Ac
torio Th
proattinio Pa
uranio U
nettunio Np
plutonio Pu
americio Am
curio Cm
berkelio Bk
californio Cf
einsteinio Es
fermio Fm
mendelevio Md
nobelio No
laurenzio Lr

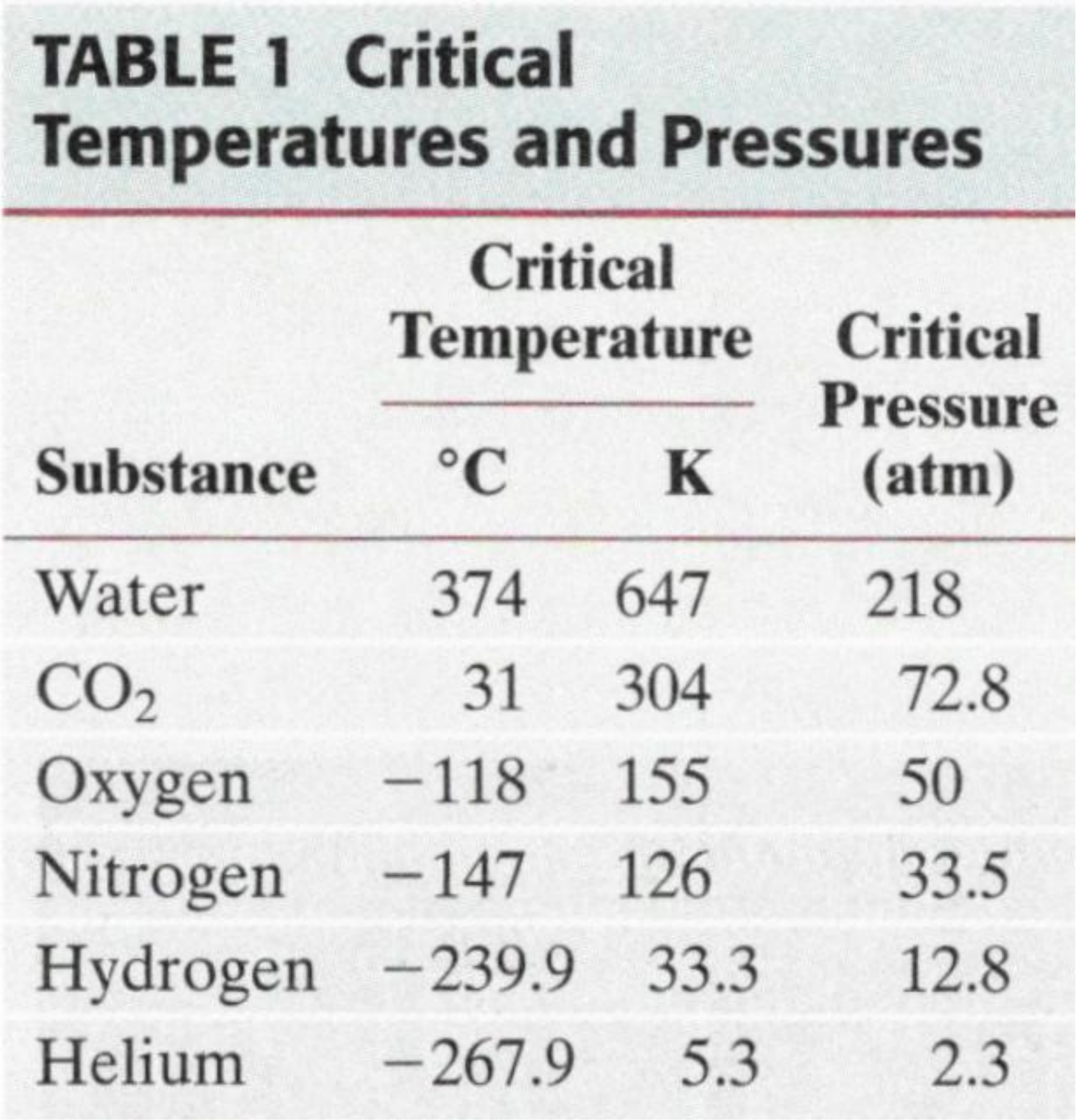


TABLE 1 Critical Temperatures and Pressures

	Critical Temperature		Critical Pressure (atm)
Substance	**°C**	**K**	
Water	374	647	218
CO_2	31	304	72.8
Oxygen	−118	155	50
Nitrogen	−147	126	33.5
Hydrogen	−239.9	33.3	12.8
Helium	−267.9	5.3	2.3

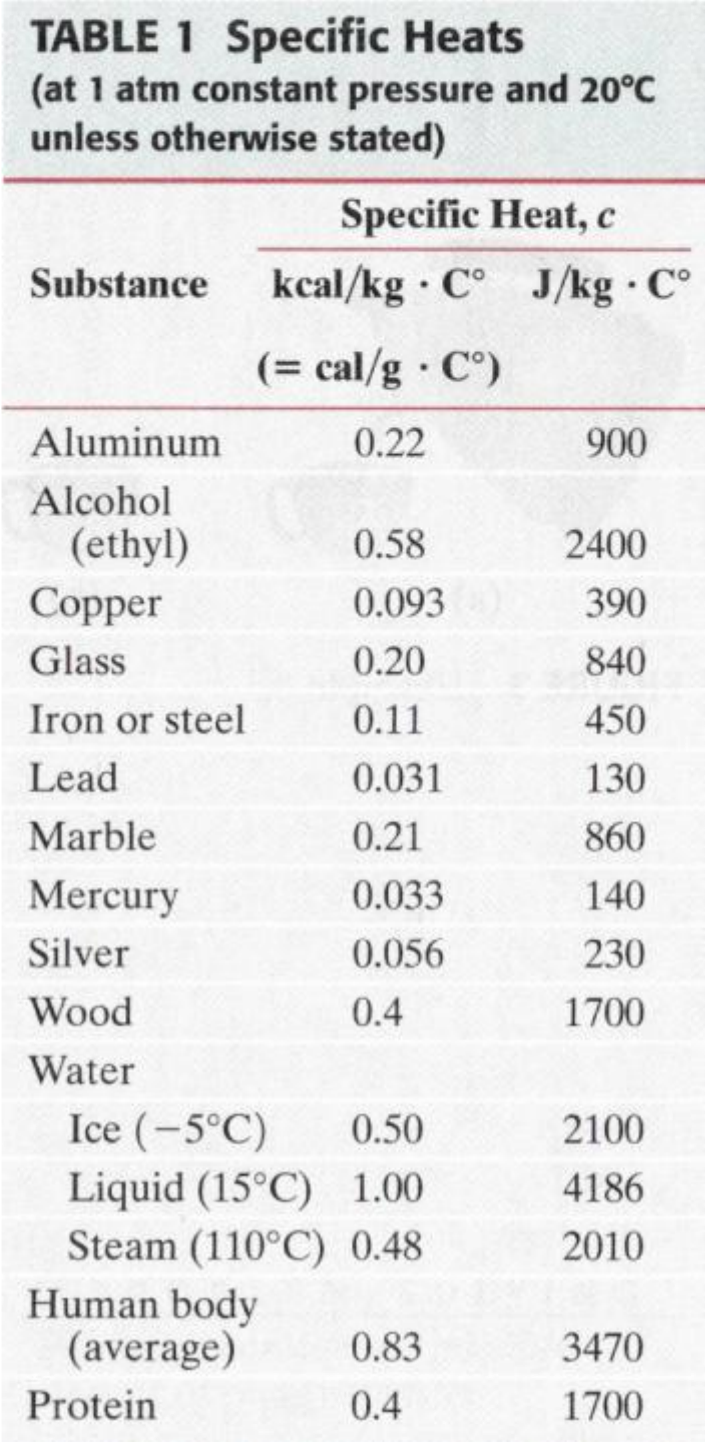


TABLE 1 Specific Heats
(at 1 atm constant pressure and 20°C unless otherwise stated)

Substance	Specific Heat, c kcal/kg · C° (= cal/g · C°)	Specific Heat, c J/kg · C°
Aluminum	0.22	900
Alcohol (ethyl)	0.58	2400
Copper	0.093	390
Glass	0.20	840
Iron or steel	0.11	450
Lead	0.031	130
Marble	0.21	860
Mercury	0.033	140
Silver	0.056	230
Wood	0.4	1700
Water		
Ice (−5°C)	0.50	2100
Liquid (15°C)	1.00	4186
Steam (110°C)	0.48	2010
Human body (average)	0.83	3470
Protein	0.4	1700

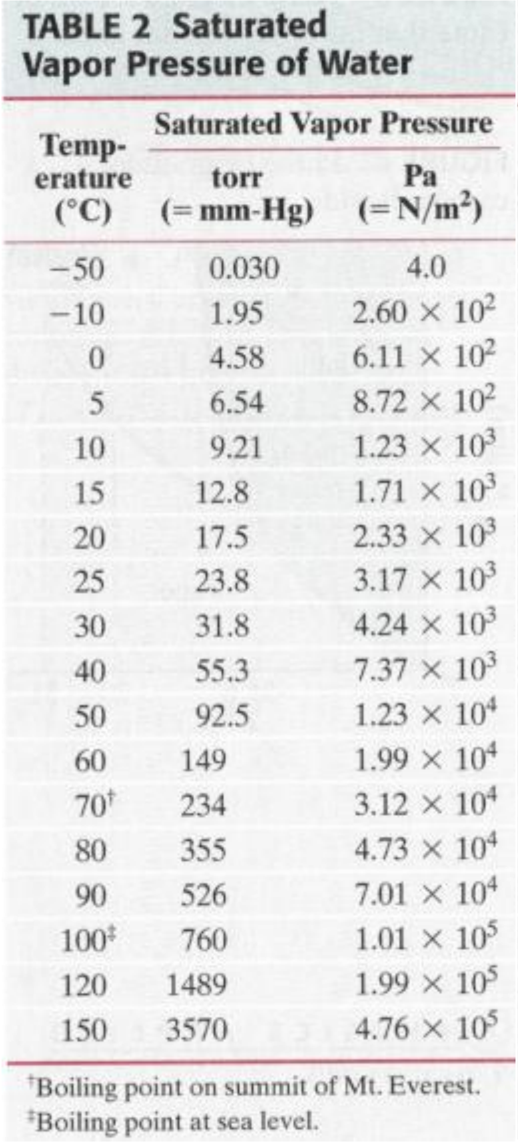


TABLE 2 Saturated Vapor Pressure of Water

Temperature (°C)	Saturated Vapor Pressure torr (= mm-Hg)	Saturated Vapor Pressure Pa (= N/m²)
−50	0.030	4.0
−10	1.95	2.60×10^2
0	4.58	6.11×10^2
5	6.54	8.72×10^2
10	9.21	1.23×10^3
15	12.8	1.71×10^3
20	17.5	2.33×10^3
25	23.8	3.17×10^3
30	31.8	4.24×10^3
40	55.3	7.37×10^3
50	92.5	1.23×10^4
60	149	1.99×10^4
70[†]	234	3.12×10^4
80	355	4.73×10^4
90	526	7.01×10^4
100[‡]	760	1.01×10^5
120	1489	1.99×10^5
150	3570	4.76×10^5

[†] Boiling point on summit of Mt. Everest.

[‡] Boiling point at sea level.

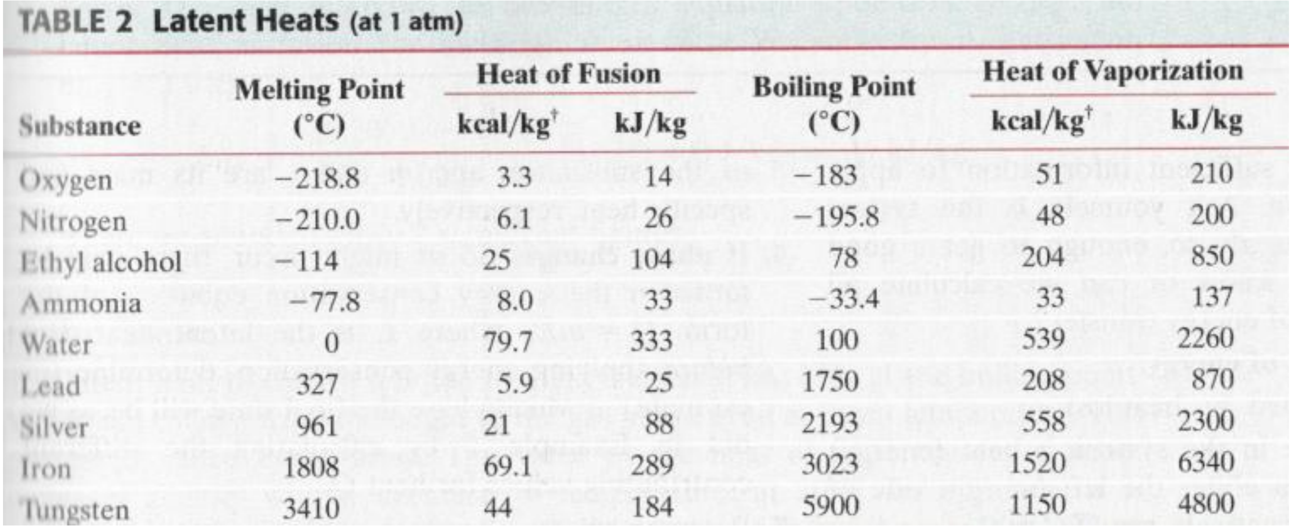


TABLE 2 Latent Heats (at 1 atm)

Substance	Melting Point (°C)	Heat of Fusion		Boiling Point (°C)	Heat of Vaporization	
		kcal/kg†	kJ/kg		kcal/kg†	kJ/kg
Oxygen	−218.8	3.3	14	−183	51	210
Nitrogen	−210.0	6.1	26	−195.8	48	200
Ethyl alcohol	−114	25	104	78	204	850
Ammonia	−77.8	8.0	33	−33.4	33	137
Water	0	79.7	333	100	539	2260
Lead	327	5.9	25	1750	208	870
Silver	961	21	88	2193	558	2300
Iron	1808	69.1	289	3023	1520	6340
Tungsten	3410	44	184	5900	1150	4800

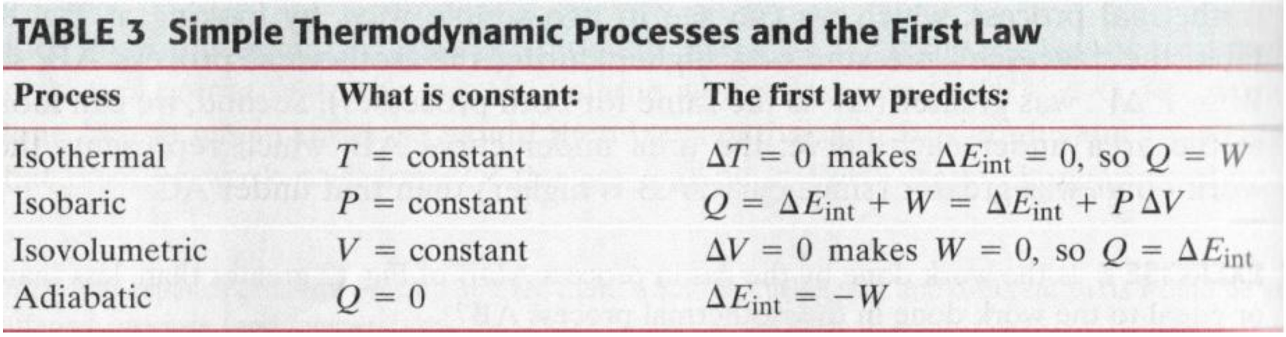


TABLE 3 Simple Thermodynamic Processes and the First Law

Process	What is constant:	The first law predicts:
Isothermal	$T =$ constant	$\Delta T = 0$ makes $\Delta E_{\text{int}} = 0$, so $Q = W$
Isobaric	$P =$ constant	$Q = \Delta E_{\text{int}} + W = \Delta E_{\text{int}} + P\,\Delta V$
Isovolumetric	$V =$ constant	$\Delta V = 0$ makes $W = 0$, so $Q = \Delta E_{\text{int}}$
Adiabatic	$Q = 0$	$\Delta E_{\text{int}} = -W$

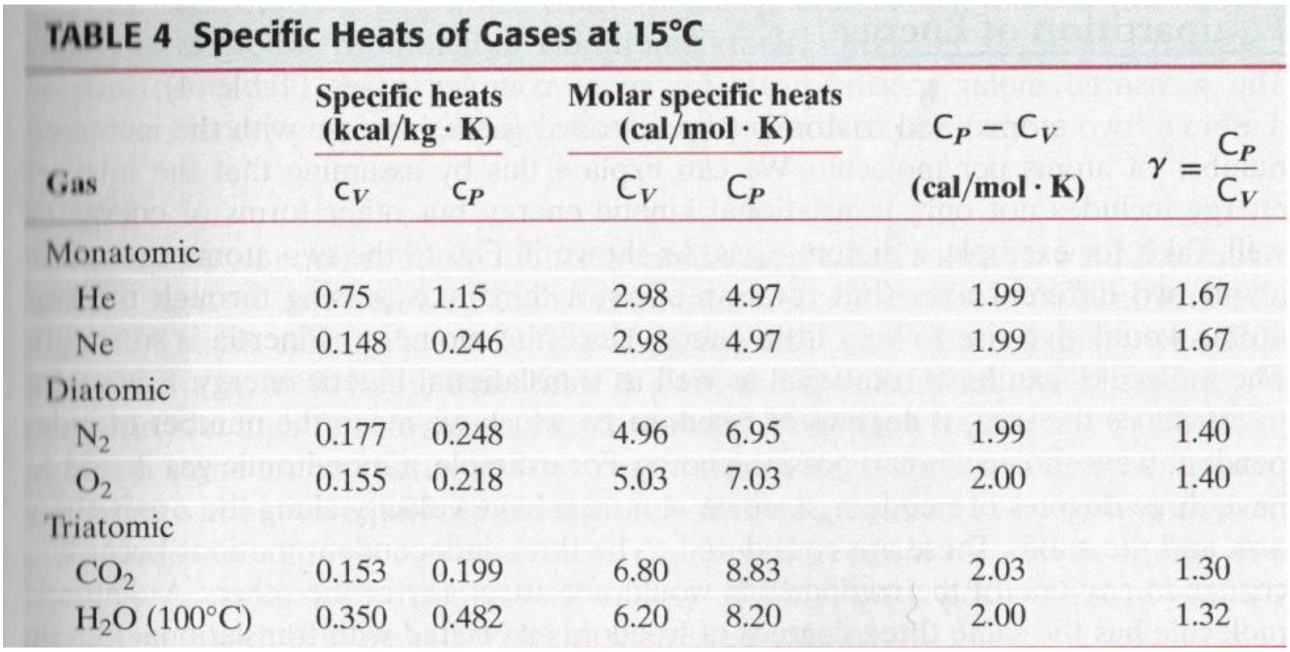


TABLE 4 Specific Heats of Gases at 15°C

Gas	Specific heats (kcal/kg · K) c_V	Specific heats (kcal/kg · K) c_P	Molar specific heats (cal/mol · K) C_V	Molar specific heats (cal/mol · K) C_P	$C_P - C_V$ (cal/mol · K)	$\gamma = \frac{C_P}{C_V}$
Monatomic						
He	0.75	1.15	2.98	4.97	1.99	1.67
Ne	0.148	0.246	2.98	4.97	1.99	1.67
Diatomic						
N_2	0.177	0.248	4.96	6.95	1.99	1.40
O_2	0.155	0.218	5.03	7.03	2.00	1.40
Triatomic						
CO_2	0.153	0.199	6.80	8.83	2.03	1.30
H_2O (100°C)	0.350	0.482	6.20	8.20	2.00	1.32

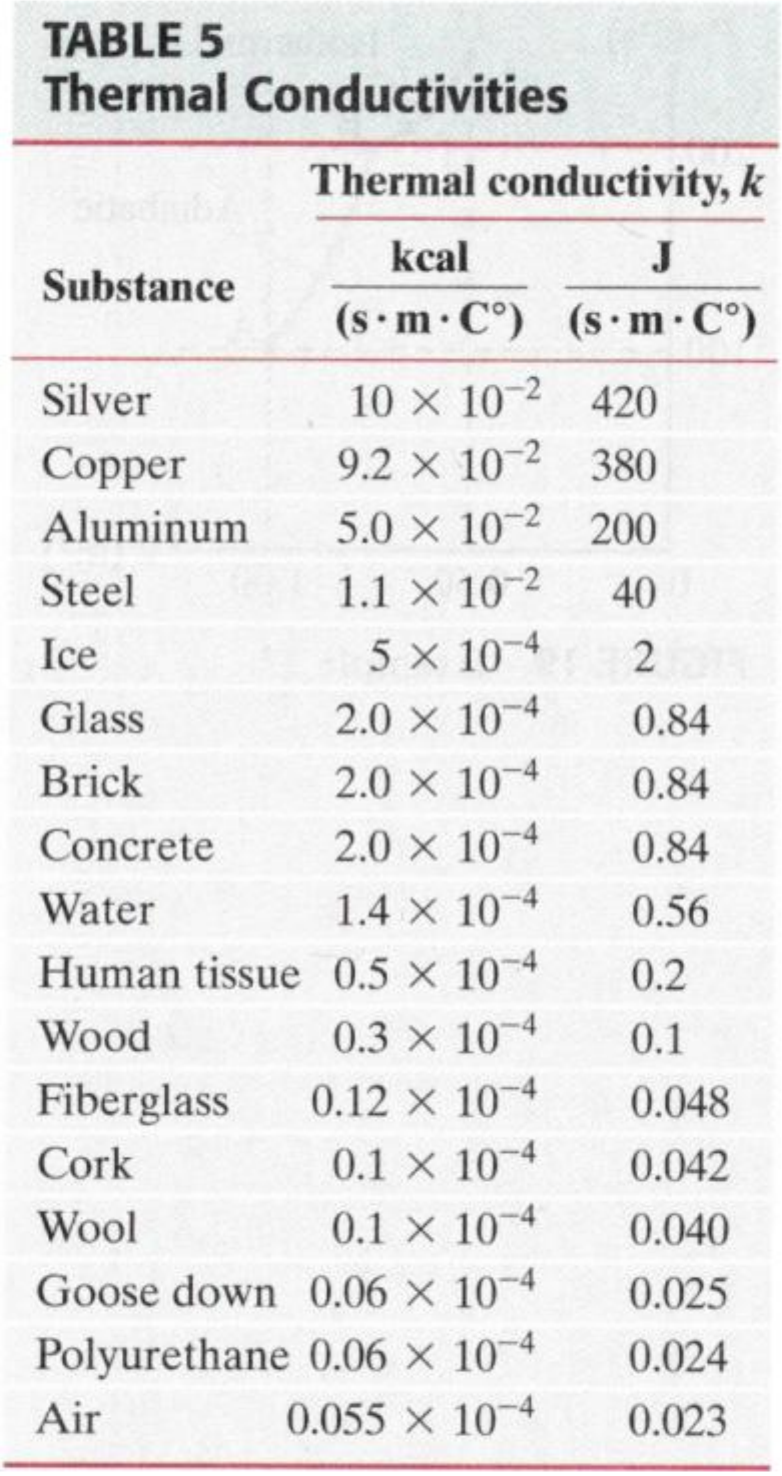
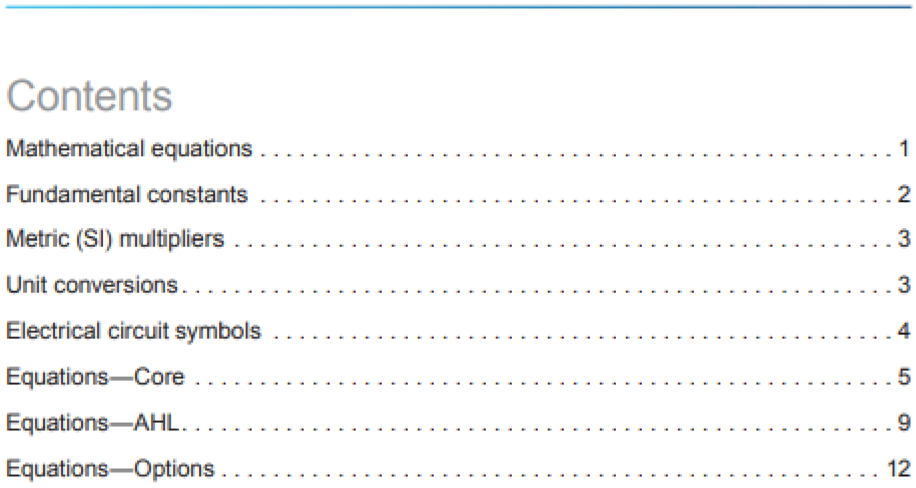


TABLE 5
Thermal Conductivities

Substance	Thermal conductivity, k	
	$\frac{\text{kcal}}{(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}$	$\frac{\text{J}}{(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}$
Silver	10×10^{-2}	420
Copper	9.2×10^{-2}	380
Aluminum	5.0×10^{-2}	200
Steel	1.1×10^{-2}	40
Ice	5×10^{-4}	2
Glass	2.0×10^{-4}	0.84
Brick	2.0×10^{-4}	0.84
Concrete	2.0×10^{-4}	0.84
Water	1.4×10^{-4}	0.56
Human tissue	0.5×10^{-4}	0.2
Wood	0.3×10^{-4}	0.1
Fiberglass	0.12×10^{-4}	0.048
Cork	0.1×10^{-4}	0.042
Wool	0.1×10^{-4}	0.040
Goose down	0.06×10^{-4}	0.025
Polyurethane	0.06×10^{-4}	0.024
Air	0.055×10^{-4}	0.023



Contents

Mathematical equations

Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Surface area of a sphere	$A = 4\pi r^2$, where r is the radius
Volume of a sphere	$V = \frac{4}{3}\pi r^3$, where r is the radius

Fundamental constants

Quantity	Symbol	Approximate value
Acceleration of free fall (Earth's surface)	g	$9.81\,\mathrm{m\,s^{-2}}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Avogadro's constant	N_A	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Boltzmann's constant	k_B	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan–Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Coulomb constant	k	$8.99 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$
Speed of light in vacuum	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass	m_e	$9.110 \times 10^{-31}\,\mathrm{kg} = 0.000549\,\mathrm{u} = 0.511\,\mathrm{MeV\,c^{-2}}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 1.007276\,\mathrm{u} = 938\,\mathrm{MeV\,c^{-2}}$
Neutron rest mass	m_n	$1.675 \times 10^{-27}\,\mathrm{kg} = 1.008665\,\mathrm{u} = 940\,\mathrm{MeV\,c^{-2}}$
Unified atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$
Solar constant	S	$1.36 \times 10^{3}\,\mathrm{W\,m^{-2}}$
Fermi radius	R_0	$1.20 \times 10^{-15}\,\mathrm{m}$

Metric (SI) multipliers

Prefix	Abbreviation	Value
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Unit conversions

1 radian (rad) $= \frac{180^\circ}{\pi}$

Temperature (K) = temperature (°C) + 273

1 light year (ly) $= 9.46 \times 10^{15}$ m

1 parsec (pc) = 3.26 ly

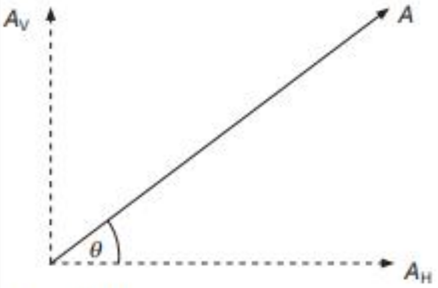
1 astronomical unit (AU) $= 1.50 \times 10^{11}$ m

1 kilowatt-hour (kWh) $= 3.60 \times 10^{6}$ J

$hc = 1.99 \times 10^{-25}\ \text{J m} = 1.24 \times 10^{-6}\ \text{eV m}$

Equations—Core

Note: All equations relate to the magnitude of the quantities only. Vector notation has not been used.

Sub-topic 1.2 – Uncertainties and errors	Sub-topic 1.3 – Vectors and scalars
If: $y = a \pm b$ then: $\Delta y = \Delta a + \Delta b$ If: $y = \frac{ab}{c}$ then: $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$ If: $y = a^n$ then: $\frac{\Delta y}{y} = \left\lvert n\frac{\Delta a}{a}\right\rvert$	 $A_H = A\cos\theta$ $A_V = A\sin\theta$

Sub-topic 2.1 – Motion	Sub-topic 2.2 – Forces
$v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{(v+u)t}{2}$	$F = ma$ $F_f \leq \mu_s R$ $F_f = \mu_d R$
Sub-topic 2.3 – Work, energy and power	**Sub-topic 2.4 – Momentum and impulse**
$W = Fs\cos\theta$ $E_K = \frac{1}{2}mv^2$ $E_p = \frac{1}{2}k\Delta x^2$ $\Delta E_p = mg\Delta h$ power $= Fv$ efficiency $= \frac{\text{useful work out}}{\text{total work in}}$ $= \frac{\text{useful power out}}{\text{total power in}}$	$p = mv$ $F = \frac{\Delta p}{\Delta t}$ $E_K = \frac{p^2}{2m}$ impulse $= F\Delta t = \Delta p$

Sub-topic 3.1 – Thermal concepts	Sub-topic 3.2 – Modelling a gas
$Q = mc\Delta T$ $Q = mL$	$p = \frac{F}{A}$ $n = \frac{N}{N_A}$ $pV = nRT$ $\bar{E}_K = \frac{3}{2}k_B T = \frac{3}{2}\frac{R}{N_A}T$

<table>
<tr><th>Sub-topic 4.1 – Oscillations</th><th>Sub-topic 4.4 – Wave behaviour</th></tr>
<tr><td>$T = \frac{1}{f}$</td><td rowspan="5">$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$
$s = \frac{\lambda D}{d}$
Constructive interference:
path difference $= n\lambda$
Destructive interference:
path difference $= \left(n + \frac{1}{2}\right)\lambda$</td></tr>
<tr><th>Sub-topic 4.2 – Travelling waves</th></tr>
<tr><td>$c = f\lambda$</td></tr>
<tr><th>Sub-topic 4.3 – Wave characteristics</th></tr>
<tr><td>$I \propto A^2$
$I \propto x^{-2}$
$I = I_0 \cos^2\theta$</td></tr>
</table>

Sub-topic 5.1 – Electric fields	Sub-topic 5.2 – Heating effect of electric currents
$I = \frac{\Delta q}{\Delta t}$ $F = k\frac{q_1 q_2}{r^2}$ $k = \frac{1}{4\pi\varepsilon_0}$ $V = \frac{W}{q}$ $E = \frac{F}{q}$ $I = nAvq$	Kirchhoff's circuit laws: $\Sigma V = 0$ (loop) $\Sigma I = 0$ (junction) $R = \frac{V}{I}$ $P = VI = I^2R = \frac{V^2}{R}$ $R_{\text{total}} = R_1 + R_2 + \ldots$ $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$ $\rho = \frac{RA}{L}$
Sub-topic 5.3 – Electric cells	**Sub-topic 5.4 – Magnetic effects of electric currents**
$\varepsilon = I(R + r)$	$F = qvB\sin\theta$ $F = BIL\sin\theta$

Sub-topic 6.1 – Circular motion	Sub-topic 6.2 – Newton's law of gravitation
$v = \omega r$ $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ $F = \frac{mv^2}{r} = m\omega^2 r$	$F = G\frac{Mm}{r^2}$ $g = \frac{F}{m}$ $g = G\frac{M}{r^2}$

Sub-topic 7.1 – Discrete energy and radioactivity

$E = hf$

$\lambda = \frac{hc}{E}$

Sub-topic 7.2 – Nuclear reactions

$\Delta E = \Delta mc^2$

Sub-topic 7.3 – The structure of matter

Charge	Quarks			Baryon number
$\frac{2}{3}e$	u	c	t	$\frac{1}{3}$
$-\frac{1}{3}e$	d	s	b	$\frac{1}{3}$

All quarks have a strangeness number of 0 except the strange quark that has a strangeness number of –1

Charge	Leptons		
–1	e	μ	τ
0	ν_e	ν_μ	ν_τ

All leptons have a lepton number of 1 and antileptons have a lepton number of –1

	Gravitational	Weak	Electromagnetic	Strong
Particles experiencing	All	Quarks, leptons	Charged	Quarks, gluons
Particles mediating	Graviton	W^+, W^-, Z^0	γ	Gluons

Sub-topic 8.1 – Energy sources

$\text{power} = \frac{\text{energy}}{\text{time}}$

$\text{power} = \frac{1}{2}A\rho v^3$

Sub-topic 8.2 – Thermal energy transfer

$P = e\sigma AT^4$

$\lambda_{\text{max}}(\text{metres}) = \frac{2.90 \times 10^{-3}}{T(\text{kelvin})}$

$I = \frac{\text{power}}{A}$

$\text{albedo} = \frac{\text{total scattered power}}{\text{total incident power}}$

Equations—AHL

Sub-topic 9.1 – Simple harmonic motion	Sub-topic 9.2 – Single-slit diffraction
$\omega = \frac{2\pi}{T}$	$\theta = \frac{\lambda}{b}$
$a = -\omega^2 x$	**Sub-topic 9.3 – Interference**
$x = x_0 \sin \omega t$; $x = x_0 \cos \omega t$	$n\lambda = d \sin\theta$
$v = \omega x_0 \cos \omega t$; $v = -\omega x_0 \sin \omega t$	Constructive interference: $2dn = \left(m + \frac{1}{2}\right)\lambda$
$v = \pm\omega\sqrt{(x_0{}^2 - x^2)}$	Destructive interference: $2dn = m\lambda$
$E_K = \frac{1}{2} m\omega^2 (x_0{}^2 - x^2)$	
$E_T = \frac{1}{2} m\omega^2 x_0{}^2$	
pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$	
mass-spring: $T = 2\pi\sqrt{\frac{m}{k}}$	
Sub-topic 9.4 – Resolution	**Sub-topic 9.5 – Doppler effect**
$\theta = 1.22\frac{\lambda}{b}$	Moving source: $f' = f\left(\frac{v}{v \pm u_s}\right)$
$R = \frac{\lambda}{\Delta\lambda} = mN$	Moving observer: $f' = f\left(\frac{v \pm u_o}{v}\right)$
	$\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$

<table>
<tr><th>Sub-topic 10.1 – Describing fields</th><th>Sub-topic 10.2 – Fields at work</th></tr>
<tr>
<td>$W = q\Delta V_e$

$W = m\Delta V_g$</td>
<td>
<table>
<tr><td>$V_g = -\frac{GM}{r}$</td><td>$V_e = \frac{kQ}{r}$</td></tr>
<tr><td>$g = -\frac{\Delta V_g}{\Delta r}$</td><td>$E = -\frac{\Delta V_e}{\Delta r}$</td></tr>
<tr><td>$E_p = mV_g = -\frac{GMm}{r}$</td><td>$E_p = qV_e = \frac{kQq}{r}$</td></tr>
<tr><td>$F_g = \frac{GMm}{r^2}$</td><td>$F_e = \frac{kQq}{r^2}$</td></tr>
</table>
<hr>
$v_{esc} = \sqrt{\frac{2GM}{r}}$

$v_{orbit} = \sqrt{\frac{GM}{r}}$
</td>
</tr>
</table>

Sub-topic 11.1 – Electromagnetic induction	Sub-topic 11.3 – Capacitance
$\Phi = BA\cos\theta$ $\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$ $\varepsilon = Bvl$ $\varepsilon = BvlN$	$C = \frac{q}{V}$ $C_{\text{parallel}} = C_1 + C_2 + \ldots$ $\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$
Sub-topic 11.2 – Power generation and transmission	$C = \varepsilon\frac{A}{d}$
$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ $R = \frac{V_0}{I_0} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$ $P_{\max} = I_0V_0$ $\bar{P} = \frac{1}{2}I_0V_0$ $\frac{\varepsilon_{\text{p}}}{\varepsilon_{\text{s}}} = \frac{N_{\text{p}}}{N_{\text{s}}} = \frac{I_{\text{s}}}{I_{\text{p}}}$	$E = \frac{1}{2}CV^2$ $\tau = RC$ $q = q_0e^{-\frac{t}{\tau}}$ $I = I_0e^{-\frac{t}{\tau}}$ $V = V_0e^{-\frac{t}{\tau}}$

Sub-topic 12.1 – The interaction of matter with radiation	Sub-topic 12.2 – Nuclear physics
$E = hf$ $E_{\max} = hf - \Phi$ $E = -\frac{13.6}{n^2}\,\text{eV}$ $mvr = \frac{nh}{2\pi}$ $P(r) = \lvert\psi\rvert^2\,\Delta V$ $\Delta x\Delta p \geq \frac{h}{4\pi}$ $\Delta E\Delta t \geq \frac{h}{4\pi}$	$R = R_0A^{\frac{1}{3}}$ $N = N_0e^{-\lambda t}$ $A = \lambda N_0e^{-\lambda t}$ $\sin\theta \approx \frac{\lambda}{D}$

Equations—Options

<table>
<tr><th>Sub-topic A.1 – The beginnings of relativity</th><th>Sub-topic A.2 – Lorentz transformations</th></tr>
<tr><td>$x' = x - vt$
$u' = u - v$</td><td rowspan="3">$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
$x' = \gamma(x - vt);\ \Delta x' = \gamma(\Delta x - v\Delta t)$
$t' = \gamma\left(t - \frac{vx}{c^2}\right);\ \Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$
$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$
$\Delta t = \gamma \Delta t_0$
$L = \frac{L_0}{\gamma}$
$(ct')^2 - (x')^2 = (ct)^2 - (x)^2$</td></tr>
<tr><th>Sub-topic A.3 – Spacetime diagrams</th></tr>
<tr><td>$\theta = \tan^{-1}\left(\frac{v}{c}\right)$</td></tr>
<tr><th>Sub-topic A.4 – Relativistic mechanics (HL only)</th><th>Sub-topic A.5 – General relativity (HL only)</th></tr>
<tr><td>$E = \gamma m_0 c^2$
$E_0 = m_0 c^2$
$E_K = (\gamma - 1) m_0 c^2$
$p = \gamma m_0 v$
$E^2 = p^2 c^2 + m_0^2 c^4$
$qV = \Delta E_K$</td><td>$\frac{\Delta f}{f} = \frac{g\Delta h}{c^2}$
$R_s = \frac{2GM}{c^2}$
$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$</td></tr>
</table>

Sub-topic B.1 – Rigid bodies and rotational dynamics	Sub-topic B.2 – Thermodynamics
$\Gamma = Fr\sin\theta$ $I = \sum mr^2$ $\Gamma = I\alpha$ $\omega = 2\pi f$ $\omega_f = \omega_i + \alpha t$ $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\theta = \omega_i t + \frac{1}{2}\alpha t^2$ $L = I\omega$ $E_{K_{rot}} = \frac{1}{2}I\omega^2$	$Q = \Delta U + W$ $U = \frac{3}{2}nRT$ $\Delta S = \frac{\Delta Q}{T}$ $pV^{\frac{5}{3}} =$ constant (for monatomic gases) $W = p\Delta V$ $\eta = \frac{\text{useful work done}}{\text{energy input}}$ $\eta_{\text{Carnot}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$
Sub-topic B.3 – Fluids and fluid dynamics (HL only)	**Sub-topic B.4 – Forced vibrations and resonance (HL only)**
$B = \rho_f V_f g$ $P = P_0 + \rho_f g d$ $Av =$ constant $\frac{1}{2}\rho v^2 + \rho g z + p =$ constant $F_D = 6\pi\eta r v$ $R = \frac{v r \rho}{\eta}$	$Q = 2\pi\frac{\text{energy stored}}{\text{energy dissipated per cycle}}$ $Q = 2\pi \times \text{resonant frequency} \times \frac{\text{energy stored}}{\text{power loss}}$

Sub-topic C.1 – Introduction to imaging	Sub-topic C.2 – Imaging instrumentation
$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$ $P=\frac{1}{f}$ $m=\frac{h_i}{h_o}=-\frac{v}{u}$ $M=\frac{\theta_i}{\theta_o}$ $M_{\text{near point}}=\frac{D}{f}+1;\ M_{\text{infinity}}=\frac{D}{f}$	$M=\frac{f_o}{f_e}$ **Sub-topic C.3 – Fibre optics** $n=\frac{1}{\sin c}$ $\text{attenuation}=10\log\frac{I}{I_0}$ **Sub-topic C.4 – Medical imaging (HL only)** $L_I=10\log\frac{I_1}{I_0}$ $I=I_0e^{-\mu x}$ $\mu x_{\frac{1}{2}}=\ln 2$ $Z=\rho c$

Sub-topic D.1 – Stellar quantities	Sub-topic D.2 – Stellar characteristics and stellar evolution
$d\,(\text{parsec})=\frac{1}{p\,(\text{arc-second})}$ $L=\sigma AT^4$ $b=\frac{L}{4\pi d^2}$	$\lambda_{\max}T=2.9\times10^{-3}\ \text{mK}$ $L\propto M^{3.5}$
Sub-topic D.3 – Cosmology	**Sub-topic D.5 – Further cosmology (HL only)**
$z=\frac{\Delta\lambda}{\lambda_0}\approx\frac{v}{c}$ $z=\frac{R}{R_0}-1$ $v=H_0d$ $T\approx\frac{1}{H_0}$	$v=\sqrt{\frac{4\pi G\rho}{3}}r$ $\rho_c=\frac{3H^2}{8\pi G}$